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Incorporating work zone impact on traffic into network-level pavement management systems

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Abstract

Nowadays, Transportation system provides a necessary linkage between people and the society, which is one of the most demanded services and it shapes people's daily living by bringing convenience. Therefore, people are considering in taking account of user cost while traveling. As the pavement condition directly affects the user cost and comfortability of drivers, pavement rehabilitation and maintenance are unavoidable in order to preserve serviceability of the roadway. Occupancy of travel lanes during rehabilitation is an ordinary situation, but it imposes extra cost on the users due to traffic disturbances. As a result, it is necessary to minimize the cost owing to rehabilitation activities.

Optimization problem regarding the agency cost due to pavement maintenance and rehabilitation activities has been widely addressed by a numerous researcher and included in the pavement management system (PMS). Although there are also many researching articles inclusive of similar optimization problems regarding the user cost due to roadway closure, their solutions are limited by either network properties underestimation or inefficient computation even if network topology is being considered. In fact, there is research showed the user cost is much higher than that of agency cost, it is crucial to quantify the traffic disturbances efficiently and include it into PMS. Therefore, optimization problem in minimizing user cost, which inclusive of additional user travel cost owing to resurfacing and additional operational cost experienced by users because of the low fuel efficiency resulting from high pavement roughness, within a network-level pavement system due to complete or partial roadway resurfacing is proposed to be dealt in this research.

This report performs a preliminary simple case on studying user cost optimization problem while considering both transportation network topology and traffic redistribution. The computation result satisfies the Pareto Optimality and it shows the Pareto set which could help to make focused tradeoffs within the related parameters.

This report consists of five chapters, including introduction and objective, background, methodologies, a case study showing solution procedures and results, and conclusion. References and Appendix for detail steps in solving the optimization problem and results would also be provided at the end of this report.



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Chapter 1: Introduction and Objective

1.1 Introduction

The transportation system is an integral section of a country development that overcomes the separation as an enabler of economic and social activities. Therefore, a reasonable cost and cozy experience of mobility are one of the considerations in our daily life. As the continuously and frequently utilization of pavement, deterioration of pavement exists and users experience a decrease of comfortability. Consequently, transportation agencies are nowadays shifting their focus from constructing a new roadway to rehabilitating and improving the current facilities. In general, pavement maintenance and rehabilitation activities are carried out to preserve the roadway's serviceability, such as resurfacing, which requires roadway closure. Occupancy of roadway lanes is an ordinary situation for maintaining the well-being of road infrastructure by undertaking rehabilitation or reconstruction. However, it imposes extra user cost on the route with heavy travel demand resulting in a traffic slowdown or even shut down, which is not welcomed by users.

Traffic congestion, either recurring or non-recurring delay, occurs when the travel demand exceeds the roadway capacity. Non-recurring congestion happens because of the incidents while the other one is caused by geometric conditions and it usually happens at bottlenecks. According to the result of a released study by Chin [1], at least 24 percent of non-recurring delay on the freeways is because of the work zone area. In fact, this percentage is under evaluated as any delay on principal arterials are not taken into account. If total delay on both freeways and principal arterials are considered, the percentage of non-recurrent congestion due to work zone is estimated to be double or even more. It is not reasonable to reconstruct the existing roadway infrastructure only for solving the cost impact of work zone area. Thus, this study showed the importance of minimization of traffic disturbances, which accounted as a part of the user cost, due to roadway closure.



Partial or complete roadway closure rehabilitation often causes a significant travel delay and traffic congestion due to the limited access of road. It is a considerable challenge for minimizing the cost effect while undergoing rehabilitation. In general, cost factor in agency's side of roadway stoppage for rehabilitation activities includes construction cost which is relatively obvious and deterministic to estimate while the cost factor in user's side, which contains travel impact cost, cannot as there is lack of information and methodology compared with the former cost. In fact, a comparison between agency cost and user cost due to roadway closure is discussed by Lee et al. [2] and they showed the user cost is normally more than double the agency cost, which revealed the seriousness of user cost resulting from traffic disturbances.

Accordingly, an applicable method for the measurement and quantification of traffic disturbances due to managerial intervention is essential to the pavement management system (PMS), as an assistant of making a decision in finding optimal strategies for pavement maintenance to its serviceable condition over a given period.

In this project, the user cost, including the additional travel cost resulting from roadway closure, and the additional operational cost due to the high roughness of pavement, are considered in dealing with optimization problem of resurfacing planning. At the end of this research, related optimization problem formulation and solution method considering both transportation network topology and traffic redistribution will be executed with the preliminary model case study.

A study on a preliminary simple network model is carried out in this project for the practical purpose. In order to solve multiple facilities' maintenance and rehabilitation optimization problem, bottom-up approach is adopted in this research which is able to take heterogeneous facility-specific features. In this final report, related solution methodology for upper-level and bottom-level optimization problems have been performed based on the study of the preliminary tiny network model.



This report consist of five chapters, introduction and objective regarding this project are listed in chapter 1. Followed by chapter 1, literature background of related optimization approaches and methodologies used in this research are discussed in chapter 2 and 3 respectively. The corresponding results on a case study with graphs and tables, including discussions based on the results, will be shown in chapter 4 while conclusion will be in the final chapter, chapter 5. Reference list and Appendix are also provided at the end of this report.

1.2 Objective

Resurfacing activities improve the current pavement condition by reducing its roughness which provides a better ride quality. Pavement with abnormal roughness would result an additional operational cost due to the low fuel efficiency and additional travel cost would be imposed owing to work zone area.

This project aims to minimize the user cost, including additional operational cost and additional travel cost, within a network level system due to a partial roadway closure resurfacing with continuous time and pavement state over an infinite time horizon by a case study in preliminary network model. Problem formulation and solution method for quantifying traffic disturbances would be executed by taking both transportation network topology and traffic redistribution into consideration. This final report would focus on a simple case study with a preliminary network model for a practical purpose in order to comprehend and evaluate the formulation and solution throughout the research that can make this study be more reliable iteration process. At the end of this research, a preliminary model for optimal resurfacing planning would be developed.



Chapter 2: Background

2.1 Literature Review

Optimization problem in maintenance and rehabilitation planning have been widely studied in literature and various solving methods are developed because of the increasing manipulation of pavement management system by transportation agencies. The focus of this project on pavement rehabilitation is specifically single resurfacing activity only. Besides, a number of researchers have addressed similar user cost minimization problems in roadway resurfacing. For example, Friesz and Fernandez [3], [4] firstly tackled this kind of problem in finding an optimal maintenance profile over time by using optimal control theory, and their work is further extended for solving optimal timing in highway stage construction. However, resurfacing activities resulted in some discontinuous jumps in the pavement condition trajectory in the context of continuous state formulations and its intermittent trajectory led the optimization problem to be impractical. Subsequently, Tsunokawa and Schofer [5] applied a continuous trend curve approximation for the resurfacing optimization solving which is different from the previous theoretical solving manner and it could overcome the deficiency of optimal control theory by handling the discontinuities in the pavement trajectory.

More recently, solutions regarding the same optimization problem of Tsunokawa and Schofer [5] have also been performed in two different ways later by Li and Madanat [6], and by Ouyang and Madanat [7] respectively. Without utilizing the trend curve approximation, they showed the threshold structure of the optimal pavement resurfacing plan even though for the uncertain deterioration model. Li and Madanat [6] made use of the steady-state property to obtain the solution and they revealed the immediateness of steady state in pavement management strategy after preliminary resurfacing over an infinite time horizon, while Ouyang and Madanat [7] solved it by using the calculus of variation over a finite time horizon. Sathaye and Madanat [8], [9], and Lee and Madanat [10] later widened the work by Li and Madanat [6] into a large-scale heterogeneous system.



Formulations for a pavement maintenance and rehabilitation can be classified into three categories according to the modelling characteristics in terms of time and pavement state, which are discrete time and discrete state [11], discrete time and continuous state [12], and continuous time and continuous state [5], while this project will fall into the third category. These system-level classifications are further divided into single or multiple facilities based on the problem scope, which represent segments of roadways. In order to solve multiple facilities' maintenance and rehabilitation optimization problem, either top-down approach [13] or bottom-up approach [8], [9] can be applied. The top-down algorithm is much more commonly used as it is naturally easy to solve, which presumes facilities are homogeneous with regard to deterioration process function and cost parameter [14]. On the other hand, the bottom-up approach is able to take heterogeneous facility-specific features [9], which is the shortcoming of the top-down approach.

Nevertheless, all studies presented above only based on system-level while network properties are not considered. In accordance with empirical data, Hawas [15] indicated that route decision of users is certainly influenced by pavement condition and there is a bidirectional relationship between traffic flow and pavement condition in the pavement deterioration model. Pavement deterioration depends on the traffic flow distribution while pavement condition is a significant factor to affect traffic flow. Therefore, it is advisable to consider the optimization problem in network-level instead of system-level. Recently, such network properties were incorporated into pavement management system by Ouyang [16], and by Medury and Madanat [17]. The exponential complexity with respect to a number of facilities, of the former solution approach, led a computationally inefficient while the latter approach only considered the network resilience that is different from our research scope and it is too conservative in terms of societal cost. Therefore, in order to overcome these shortcomings of previous studies, this research will consider both transportation network topology and traffic redistribution for solving user cost minimization problem due to roadway closure resurfacing.



In order to quantify pavement condition, roughness is used as the key parameter, which depends on the traffic flow and closely related to the pavement rehabilitation. Roughness is a critical pavement characteristic as it directly influences the user cost in a transportation system, especially for the additional operating cost resulting from high roughness. It is defined as the irregularities in the pavement surface while the International Roughness Index (IRI) is the standard measurement of roughness which is determined by mkm^{-1} or $inmi^{-1}$, the higher value of IRI represent the more uneven of the pavement surface. It is well known to perform pavement resurfacing when IRI reach the trigger roughness as driver begin to feel uncomfortable beyond this value while extra operating cost will be induced [18].

In short, there are several approaches can be taken as the reference in solving the optimization problem of this project, which is the minimization of travel impact due to work zone area, such as optimal control theory, continuous trend curve approximation, the use of the immediateness steady-state property of pavement and the calculus of variation etc. The approaches listed above are all restricted in system level optimization while this project focus on the whole network owing to the bidirectional relationship between traffic flow and pavement condition; therefore none of the above methods would be applied. However, this project will follow the ideas of optimality equations proposed by Sathaye and Madanat [9], and by Ouyang and Madanat [7] at the same time. Those equations would be amended correspondingly to fit our topic requirements and considerations, which cover both transportation network topology and traffic redistribution over an infinite time horizon. In addition, as this project treats every road facility to be heterogeneous with regard the deterioration process, the bottom-up algorithm is more suitable to this project compare with the top-up approach.



Chapter 3: Methodologies

This session briefly describes the development of this research and the basic principles with a summarization of the problem formulations used in different aspect, including basic formulations, cost formulations, model formulations and optimality formulations. Besides, at the end of this session, solution method is provided for solving the network level work zone impact optimization problem.

3.1 Project Development

This project is developed in several phases, including planning, design, implication, evaluation, and organization. To clarify the research topic, it is essential to have some literature review for identifying the problem and its significance, follow by setting scope and objective which specifically for this project. Once the problem is identified and settled down, some formulations, assumptions, and constraints should be made by taking references on those literature reviews which are related to the project to solve the problem. A preliminary network model and data are then generated as the case study for executing the solution. The result is checked regarding its reasonability; it would be an iteration process to reformulate and examine the constraints and model until it makes sense if the result is not reliable. Finally, the debugged result would be organized and prepared for documentation. The flow diagram of this project is shown in next page.

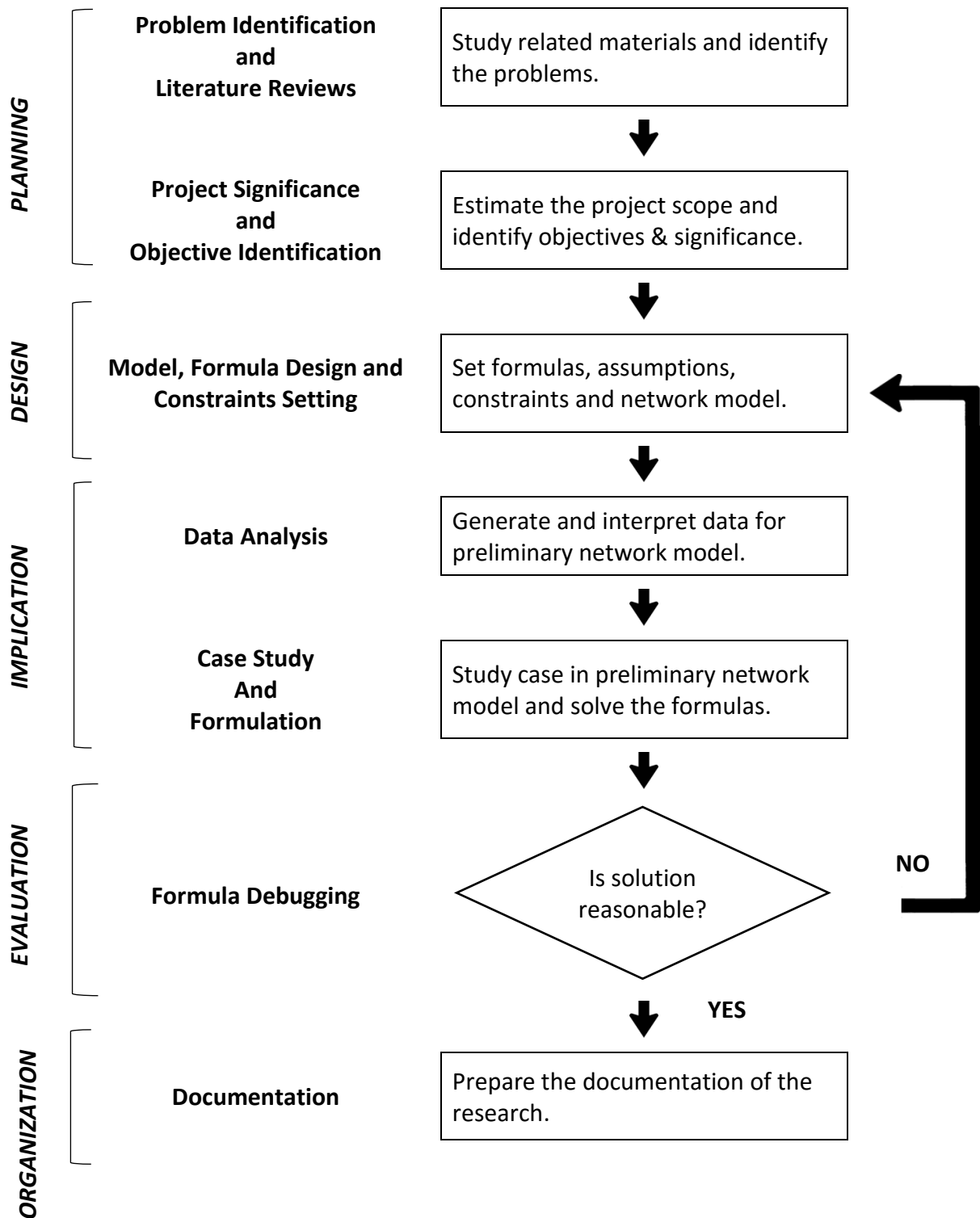


Figure 1: Flow Diagram of Project Development



3.2 Basic Principles

User cost is minimized for the single resurfacing activity within a network-level system in this research, which is considered as a combination of additional operational cost due to pavement roughness and additional travel impact cost, in term of both additional vehicle-hour-travelled (VHT) and additional vehicle-miles-travelled (VMT), owing to roadway closure resurfacing. Sathaye and Madanat [9] solved a similar problem and described a comprehensive methodology for dealing with the budget limited system-level optimization problem effectively. As the result, this research would be carried out based on what Sathaye and Madanat [9] did, however, instead of the rehabilitation cost per time for a single resurfacing activity, the agency budget constraint was replaced by a yearly available discounted budget. At the same time, solution method proposed by Ouyang and Madanat [7] would also be taken as references.

The network-related disturbances optimization problem here includes two cost factors, which are the user cost and the agency cost. User cost consists of operational cost and traffic delay impact cost while the agency cost includes the roadway maintenance cost. In this project, the traffic disturbances resulting from the roadway closure would be estimated based on user equilibrium (UE) model. In the state of user equilibrium, user travel time cannot be further reduced by unilaterally shifting route within the origin-destination (O-D) pair [20]. Furthermore, the travel time per road link is regulated consistent with the ideas of Bureau of Public Roads (BPR) link congestion function, which the travel time is assumed to be a function of traffic flow and total capacity in each road link. On the other hand, agency cost here represent resurfacing maintenance cost as this research only focus on single resurfacing activity specifically, and this cost is based on the realistic empirical models of deterioration and rehabilitation which describe the behavior of the facilities.



3.3 Basic Formulation

The basic formulation for the pavement resurfacing optimization problem in conjunction with the objective function in Eq. (1) and constraints in Eq. (2). The objective function, Eq. (1), is the summation of annual average discounted user cost for each site (\$/yr), denoted V_j , resulting from single resurfacing activity performed on site $j = 1, 2, \dots, J$, and additional roughness levels due to deterioration with respect to two decision variables, x_j and τ_j , which are the threshold roughness of pavement (IRI, m/km) and the resurfacing period (year) at site j respectively.

The overall user cost includes the additional travel cost which is in term of both distance and time resulting from roadway closure, and the additional operational cost associated with pavement condition which is indicated by the International Roughness Index (IRI). The overall user cost is then converted into annualized discounted cost by multiplying a factor $\left[\frac{e^r - 1}{1 - e^{-rx_j}} \right]$, where r is the discount rate [19], which is closely depend on the society interest model. From the formulation H_j represents the additional travel cost, which relates to additional vehicle hours travelled (VHTs) and vehicle miles travelled (VMTs) for all links included in the whole network resulting from a single resurfacing activity performed on site j , while U_j represents the discounted additional user operational cost due to pavement roughness level, which is an integral from 0 to τ_j , depends on threshold pavement roughness and resurfacing period.



The budget constraint, Eq. (2), indicates the restriction on the annual average discounted rehabilitation cost at site j , denoted A_j , while B represents the annual available discounted budget. Besides the limitation shown in Eq. (2), some network constraints, such as flow conservation and O-D demand will be also taken into consideration for this research.

$$\min_{\substack{x=\{x_j:\forall j\} \\ \tau=\{\tau_j:\forall j\}}} \sum_{j=1}^J V_j(x_j, \tau_j) = \sum_{j=1}^J \left[\frac{e^r - 1}{1 - e^{-r\tau_j}} \right] \cdot [H_j + U_j(x_j, \tau_j)] \quad \text{Eq. (1)}$$

$$s. t. \sum_{j=1}^J A_j(x_j, \tau_j) \leq B \quad \text{Eq. (2)}$$

The Lagrangian mathematics formulation method is taken for this project as it is simple to solve, the converted Lagrangian form of the basic formulation in this project is shown below in Eq. (3), which treats constraints explicitly as extra equations, where Λ is the Lagrange multiplier. The objective of this formulation is to minimize the threshold roughness of pavement and the resurfacing period while the constraints are in the maximum level.

$$\min_{x, \tau} \max_{\Lambda} L = \sum_{j=1}^J V_j + \Lambda \left[\sum_{j=1}^J A_j - B \right] \quad \text{Eq. (3)}$$



Notations Used in Basic Formulation

j	Index of links within the whole network, $j \in J$
$V_j(x_j, \tau_j)$	Annual average discounted user cost resulting from resurfacing activities performed on site j and additional roughness levels due to deterioration, (\$/yr)
x_j	Threshold roughness of pavement at site j (IRI, m/km)
τ_j	Resurfacing period at site j (year)
r	Discount rate
H_j	Annual average additional vehicle travel cost due to resurfacing activity performed on site j , including the sum of additional vehicle-hour-travelled (VHT) and vehicle-miles-travelled (VMT) of all links, (\$/yr)
$U_j(x_j, \tau_j)$	Discounted additional user operational cost due to pavement roughness, which is an integral from 0 to τ_j (\$/yr)
A_j	Annual average discounted rehabilitation cost at site j (\$/yr)
B	Annual available discounted budget for pavement maintenance activity (\$/yr)

Table 1: Descriptions of notations used in basic formulation



3.4 Cost Formulation

3.4.1 Agency Cost

Annual average discounted rehabilitation cost represents the annual resurfacing maintenance cost, which can be expressed as the function showed below (Eq. (4)) with a discount factor. It consists of a fixed set-up cost, such as labour and machines, denoted n_j , and a variable cost which is proportional to the resurfacing intensity denoted m_j . A larger amount of rehabilitation cost is required for a greater degree resurfacing intensity which result in a more effective resurfacing. Generally, the annual average discounted rehabilitation cost at site j , A_j , also includes the consideration of the number of lane to be resurfaced, l_j , and the lane distance, d_j .

$$A_j = \left[\frac{e^r - 1}{1 - e^{-r\tau_j}} \right] \left((m_j w_j + n_j) \cdot l_j \right) \cdot d_j \quad \text{Eq. (4)}$$

However, pavement should not be over resurfaced which will contribute an excessive and worthless cost as resurfacing intensity is not following the trend of more is better. The maximum effective resurfacing intensity w_{maxj} is a linear function of threshold pavement roughness, x_j , with h_j and p_j which are some rehabilitation parameters.

$$w_{maxj} = h_j x_j + p_j \quad \text{Eq. (5)}$$

Substitution w_{maxj} into Eq. (4) for w_j result in Eq. (6), which provides A_j as a function of threshold pavement roughness, x_j . The annual average discounted rehabilitation cost with the maximum resurfacing intensity is expected to be the highest pavement maintenances cost for the network.

$$A_j = \left[\frac{e^r - 1}{1 - e^{-r\tau_j}} \right] \left[(m_j (h_j x_j + p_j) + n_j) \cdot l_j \cdot d_j \right] \quad \text{Eq. (6)}$$



3.4.2 User Cost

The user travel cost, includes VHT and VMT, which is considered as a factor in this research for the optimization of rehabilitation activity. Vehicle Hours Travelled (VHT) is the total vehicle hours expended traveling on the roadway in the network system during the pavement rehabilitation period. The total hours that vehicles travelled in link j is defined as the multiplication of corresponding travel time, $T_t^{j,k}$ (hr/veh), and traffic flow, $q_t^{j,k}$ (veh/hr), at discrete time t ($t \in [0, D_k - 1]$ hr), when link k is under construction throughout the resurfacing duration, D_k (hrs).

$$VHT_j = \sum_{j=1}^J \sum_{t=0}^{D_k-1} VHT_j^k = \sum_{j=1}^J \sum_{t=0}^{D_k-1} T_t^{j,k} \cdot q_t^{j,k} \quad \text{Eq. (7)}$$

On the other hands, Vehicle Miles Travelled (VMT) is the measurement of the total miles travelled by all vehicles across the network system during the pavement rehabilitation period. Normally, it is expressed in miles/hr, but here km/hr is used as the unit. Therefore, VMT here is specifically expressed as the measurement of the total kilometre travelled by all vehicles in an hour in this research. When link k is under rehabilitation, the total VMT experienced by drivers is defined as the multiplication of total distance of link j , d_j (km), and corresponding traffic flow, $q_t^{j,k}$ (veh/hr), throughout the resurfacing duration, D_k (hrs).

$$VMT_j = \sum_{j=1}^J \sum_{t=0}^{D_k-1} VMT_j^k = \sum_{j=1}^J \sum_{t=0}^{D_k-1} d_j \cdot q_t^{j,k} \quad \text{Eq. (8)}$$



In order to find out the additional user cost, the cost comparison between the normal situation and that during the resurfacing duration should be studied. Therefore, in this research, ΔVHT_j and ΔVMT_j would be dealt. ΔVHT_j is considered for calculating the additional VHT which is the difference of vehicles' hours travelled for all link j from origin to destination between the situation that without any work zone across the network and the situation when link k is under resurfacing. The formulation can be express as in Eq. (9).

$$\Delta VHT_j = \sum_{j=1}^J \sum_{t=0}^{D_k-1} T_t^{j,k} \cdot q_t^{j,k} - \sum_{j=1}^J \sum_{t=0}^{D_k-1} T_t^j \cdot q_t^j \quad \text{Eq. (9)}$$

Similarly, ΔVMT_j is considered for calculating the additional distance travelled which is performed by replacing kilometre travelled by all vehicles to vehicle hours travelled. The Eq. (10) show the formulation of ΔVMT_j .

$$\Delta VMT_j = \sum_{j=1}^J \sum_{t=0}^{D_k-1} d_j \cdot q_t^{j,k} - \sum_{j=1}^J \sum_{t=0}^{D_k-1} d_j \cdot q_t^{j,k} \quad \text{Eq. (10)}$$



Annual average additional vehicle travel cost, H_j , is the summation of the ΔVHT s and ΔVMT s with respect to their corresponding user cost parameter, u_1 and u_2 , for all lanes across the network resulting from a resurfacing activity performed on site j , where u_1 and u_2 are the cost parameters with well-known values that can be found from empirical research.

It is conceptual that the resurfacing activity would not be carried out for the whole lane at once which will cause an excessive impact on the current traffic flow. Each lane in link k which need to be resurfaced will be divided into several sections and be resurfaced one by one. Therefore, the additional travel cost function should later multiply the total number of sections resurfaced, where d_j and z represent the total distance of lane in link j and resurfacing section length respectively.

$$H_j = (\Delta VHT_j \cdot u_1 + \Delta VMT_j \cdot u_2) \cdot \frac{d_j}{z} \quad \text{Eq. (11)}$$

The annual discounted additional user operational cost, U_j , during a resurfacing period, τ_j , due to the pavement roughness is an integral from 0 to τ_j , which is depend on the deterioration model, traffic flow and pavement condition state, where b is the rate of pavement deterioration.

$$U_j = d_j \int_0^{\tau_j} 365.24 \cdot \sum_{t=0}^{23} q_t^j \cdot c \cdot s_j^+ \cdot e^{(b-r)t} dt \quad \text{Eq. (12)}$$



Notations Used in Cost Formulation

m_j, n_j	Parameter of rehabilitation cost
l_j	Number of lane to be resurfaced
w_j	Resurfacing intensity (mm)
w_{maxj}	Maximum effective resurfacing intensity (mm)
h_j, p_j	Parameters of rehabilitation activity
k	Index of links under resurfacing $k \in J$
VHT_j	Total vehicle hours expended traveling in link j without any work zone across the network (hr·veh/hr)
VHT_j^k	Total vehicle hours expended traveling in link j when link k is under resurfacing (hr·veh/hr)
$VM T_j$	Total kilometre travelled by all vehicles in an hour in link j without any work zone across the network (km·veh/hr)
$VM T_j^k$	Total kilometre travelled by all vehicles in an hour in link j when link k is under resurfacing (km·veh/hr)
D_k	Resurfacing duration on link k (hr)
$T_t^{j,k}$	Travel time in link j when link k is under resurfacing activity at time t (hr/veh)
T_t^j	Travel time in link j without any work zone across the network at time t (hr/veh)
$q_t^{j,k}$	Traffic flow in link j when link k is under resurfacing activity at time t (veh/hr)
q_t^j	Traffic flow in link j without any work zone across the network at time t (veh/hr)
u_1, u_2	Parameter of user travel cost
d_j	Total distance of lane in link j (km)
z	Resurfacing section length (km)
c	Parameter of user operational cost (\$/veh/IRI/km)
s_j^+	Pavement roughness just after resurfacing (IRI, m/km)
b	Deterioration Rate
t	Time from 0 to the construction duration (0~1 hr, 1~2 hr, ... $D_k - 1 \sim D_k$ hr)

Table 2: Descriptions of notations used in cost formulation



3.4.3 Deterioration model and Rehabilitation Formulation

The formula of deterioration model is used to describe the deterioration behavior of pavement, a function of pavement condition just after resurfacing and the time since resurfacing. The pavement roughness is continuously increasing between two consecutive resurfacing periods, and the deterioration process is treated as the exponential function of time. The deterioration rate, b , depends on the deterioration model which is used in the research can be affected by internal or external factors, such as traffic load, climate effect, and moisture content etc. Each link is supposed to have a different value of b . The mathematic expression of pavement deterioration model is defined as below.

$$F(t_j, s_j^+) = s_j = s_j^+ \cdot e^{bt_j} \quad \text{Eq. (13)}$$

Substitution τ_j into Eq. (13) for t_j result in Eq. (14) which provides deterioration function expressed in term of resurfacing period and it also represents the threshold roughness of pavement.

$$F(\tau_j, s_j^+) = x_j = s_j^+ \cdot e^{b\tau_j} \quad \text{Eq. (14)}$$

Resurfacing effectiveness indicate the improvement of pavement condition after resurfacing which is expressed as a linear function of w_j to a certain maximum effective level G_{maxj} and G_{maxj} is a linear function of x_j in the segment-level optimality without agency budget constraint.

$$G_j(w_j, x_j) = x_j - s_j^+ = \frac{G_{maxj}w_j}{w_{maxj}} = \frac{g_1x_jw_j}{[h_jx_j + p_j]} = g_1x_j \quad \text{Eq. (15)}$$



However, if agency budget exists, there is a limitation on the resurfacing intensity where the optimal overlay thickness can be lower than the maximum effective resurfacing intensity in order to save money. Therefore, the resurfacing effectiveness function can be expressed as below.

$$G_j(w_j, x_j) = x_j - s_j^+ = g_1 x_j \cdot \min \left\{ 1, \frac{w_j}{w_{maxj}} \right\} = g_1 x_j \cdot \min \left\{ 1, \frac{w_j}{g_2 x_j + g_3} \right\} \quad \text{Eq. (16)}$$

From Eq. (16), any resurfacing intensity which is greater than $g_2 x_j + g_3$ would be suboptimal. Therefore, the resurfacing effectiveness and intensity can be rewritten into Eq. (17) and Eq. (18) due to the truncation which is practical later in solving the segment-level optimization problem.

$$G_j(w_j, x_j) = g_1 x_j \cdot \frac{w_j}{g_2 x_j + g_3} = \frac{g_1 x_j w_j}{g_2 x_j + g_3} \quad \text{Eq. (17)}$$

and the resurfacing intensity should be restricted as follow:

$$w_j \leq g_2 x_j + g_3 \quad \text{Eq. (18)}$$

Notations Used in Deterioration model and Rehabilitation Formulation

$F(t_j, s_j^+)$	Pavement deterioration model
s_j	Pavement roughness in link j (IRI, m/km)
$G_j(w_j, x_j)$	Resurfacing effectiveness model
G_{maxj}	Maximum resurfacing effectiveness
g_j	Parameter of rehabilitation activity

Table 3: Descriptions of notations used in deterioration model and rehabilitation formulation



3.5 Solution Method

In this project, a bottom-up approach is applied to solve the cost optimization problem which regarding the incorporation of the work zone impacts on traffic. Traffic flow redistribution would be carried out, and the optimal traffic flow distribution due to roadway closure is used as one of the considerations on the overall user cost minimization resulting from the resurfacing.

First of all, traffic disturbances are quantified for obtaining the optimal traffic flow distribution based on the user equilibrium model in order to generate the parameters required for computing the objective equation, as the user additional travel cost and user additional operational consist depend on the traffic flow. Once all the elements included in the objective function is ready, a network-level optimization would be carried out to obtain the optimal threshold pavement roughness, optimal resurfacing period and the optimal resurfacing intensity. Finally, the overall user cost and the rehabilitation cost resulting from the resurfacing can be executed, and the amount of the cost with respect to the corresponding optimal pavement roughness level and resurfacing period can also be drawn from their relationship by plotting a graph. The procedures are briefly showed in the following:

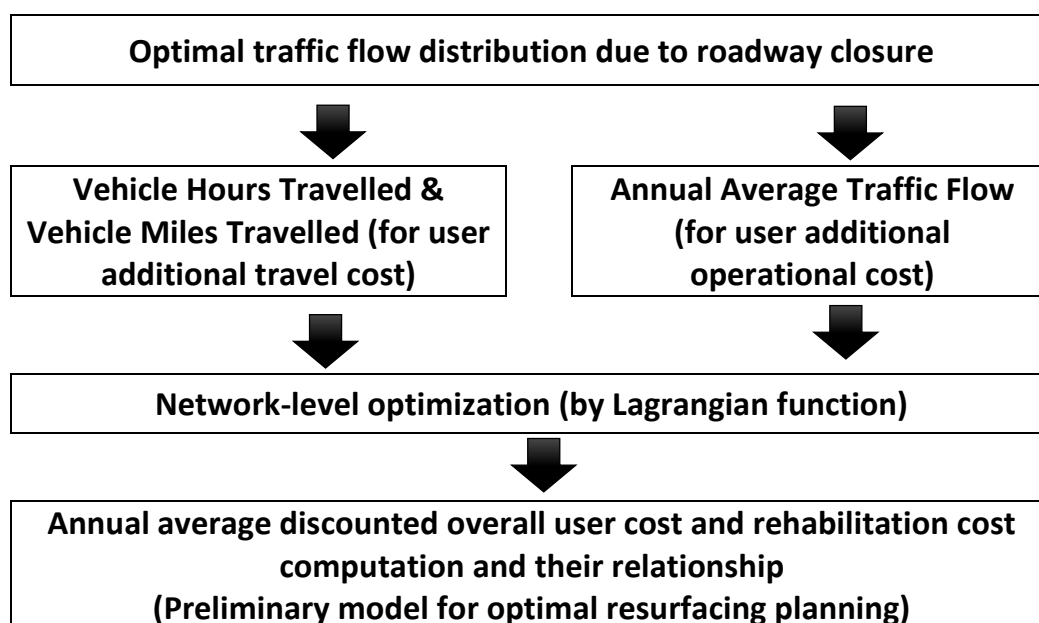


Figure 2: Outline of Solution Method

Chapter 4: Case Study - Solution Procedures and Results

A preliminary simple network model regarding to the work zone impact on traffic into network-level pavement management systems is designed for this report. As the complexity of the real-world case, it is important to create a trial study for the comprehension and evaluation of formulation throughout this research. This chapter provides the information of the network model used, the corresponding procedures, and results of traffic redistribution and optimization due to work zone impact.

4.1 Background of Preliminary Network Model

In the following session, the optimal traffic flow distribution is performed based on the user equilibrium model, and the related user travel cost and rehabilitation cost are calculated for the simple case study. The background of the preliminary network model is stated in below.

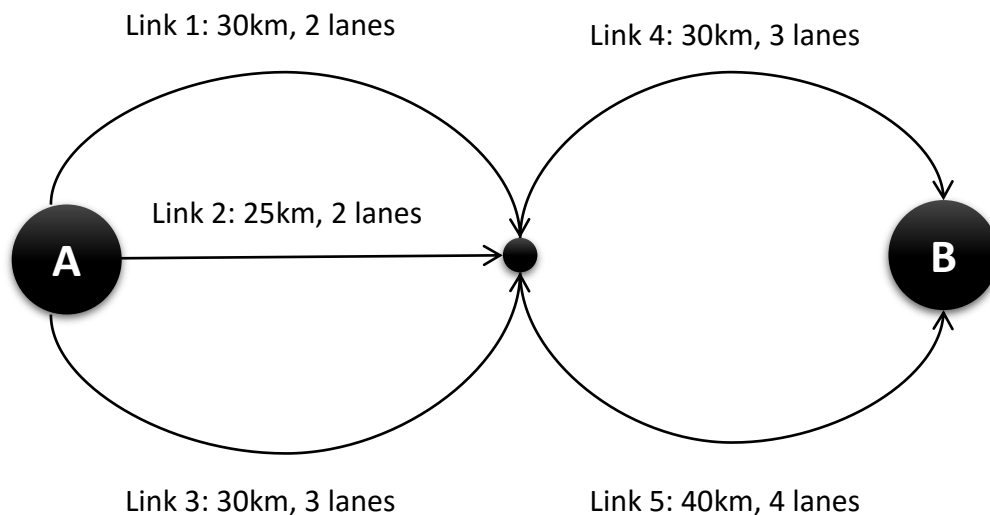


Figure 3: Situation of preliminary tiny network model



There are totally five links in the small one-way network from origin A to destination B passing through an intermediate node. Every link has different number of lane and link distance which may affect the route choice of drivers and thus the overall traffic flow distribution according to the user equilibrium model.

It is assumed that the full lane capacity is 2,000 vehicles per hour with the free flow speed of 100 km/hr. The O-D demand differ during the different period of a day, it is divided into daytime and night time and both take twelve hours each in this network study for simple calculation.

Also, it is conceptual that the O-D demand during night time is much less than that in the daytime, therefore here assume the demand in the night time is one-tenth of that in the daytime while the O-D demand between node A and B during daytime is 14,000 veh/hr.

4.2 Traffic Flow Distribution

This report would show two methods for calculating the traffic flow distribution by all-or-nothing traffic assignment with additional simple assumptions and by user equilibrium flow distribution with Bureau of Public Roads (BPR) link congestion function respectively. In general, the former method would not be taken in a research as it is too simple and result in an unrealistic flow pattern. However, it is a good practise to include it in this report as an example to compare with the latter method which would show the deficiency of minimization by the first method.



Method 1: Minimization by All-or-Nothing Traffic Assignment with Additional Simple Assumptions

The simplest route choice and assignment method is all-or-nothing assignment. This method assumes that there are no congestion effects, that all drivers consider the same attributes for route choice and that they perceive and weigh them in the same way. The traffic flow on links is assigned without consideration of whether there is adequate capacity or heavy congestion where the travel time is a fixed input and does not vary depending on the congestion on a link. Based on the all-or-nothing assignment, all drivers are assigned to one route between O-D pair and no driver is assigned to other less attractive routes. However, in this preliminary tiny network, some additional assumptions are taken into account to form a kind of more reasonable result.

The first assumption is that there is a limitation on link capacity, which means drivers are assigned to an attractive route unless the link reaches its capacity and then they will be assigned to the second attractive route, to utilize every link across the network. If links are having the same time cost, it is assumed that a higher portion of traffic flow will be assigned to the one with larger overall capacity.

There are two assignment sections within the tiny network, the first one is from origin A to an intermediate node while another one is from the intermediate node to destination B, therefore drivers are assigned separately and there should be flow conservation on overall O-D demand. During daytime, from A to the intermediate node, 4000 vehicles are assigned to link 2, which is its maximum capacity. As link 1 and 3 have the same travel distance, 40:60 split of traffic flow is performed on these two links for the remaining 10000 travel demand according to the second assumption. 6000 and 8000 vehicles are assigned to link 4 and 5 respectively from the intermediate node to B with respect to their link capacity. During night time, the O-D demand is much below to the link capacity of both section, hence all drivers are assigned to one link which has the shortest distance.



The calculation and result are shown in table below.

O-D Demand Traffic Flow	Daytime: 14,000 veh/hr	Night Time: 1,400 veh/hr
A - Intermediate node		
q₁	10000*0.4=4000 veh/hr	0 veh/hr
q₂	4000 veh/hr	1400 veh/hr
q₃	10000*0.6=6000 veh/hr	0 veh/hr
Intermediate node - B		
q₄	6000 veh/hr	1400 veh/hr
q₅	8000 veh/hr	0 veh/hr

Table 4: Computation result of traffic flow distribution by all-or-nothing traffic assignment with additional simple assumptions

From the result, it showed the traffic flow distribution in each link is within the overall link capacity. However, it is not a realistic situation as there is a potential of having more traffic flow in the link that exceeds its maximum capacity if it's time cost still less than the other link. Even though the link has already reached its capacity, it is still attractive to the drivers under the reasonable situation. The method of all-or-nothing traffic assignment with additional simple assumptions omitted the possibility of over-capacity and thus result in an impractical flow distribution. Therefore, it is not a good practise to use this method for a rational research.

In fact, this all-or-nothing assignment model is an important practical application which acts as a building block for other types of assignment techniques. Since it has a limitation which ignores the fact that the link travel time is a function of link volume when there is congestion or multiple paths are used to carry traffic, method 2 is performed in following part.



Method 2: Minimization by User Equilibrium Flow Distribution with Bureau of Public Roads (BPR) link congestion function

Under user equilibrium, which is a route choice assumption proposed by Wardrop, stated each driver chooses the best route with less travel time where vehicles are assumed of travelling at free-flow speed at the beginning.

The travel time per road link is regulated consistent with the ideas of Bureau of Public Roads (BPR) link congestion function, which the travel time is assumed to be a function of traffic flow, q , link capacity, C_j , and the travel time between O-D pair under the free flow speed condition, T_o^j , in road link j . Hence, the optimal traffic flow can be solved by minimizing the sum of travel time on all links between an origin and destination and each O-D pair can be expressed with an objective equation in Eq. (19) and constraint in Eq. (20).

The objective function, Eq. (19), is the summation of travel time for all link $j = 1, 2, \dots, J$, between an O-D pair within the network (hours), denoted $Z_{j,k}$, resulting from resurfacing activities performed on link k where the travel time is an integral from 0 to q_j . The constraints function, Eq. (20), conserve the traffic flow in an overall O-D demand. The summation of traffic flow for all link j should equal to the total O-D demand, Q .

$$\min_{q=\{q_j:\forall j\}} Z_{j,k} = \sum_{j=1}^J \int_0^{q_j} T_o^j \left(1 + 0.15 \left(\frac{q}{C_j} \right)^4 \right) dq \quad \text{Eq. (19)}$$

$$s. t. \sum_{j=1}^J q_j = Q \quad \text{Eq. (20)}$$



In general, the travel time between an O-D pair under a free flow speed condition on link j in the network, T_o^j , is calculated by dividing the total distance of link j , d_j (km), by the vehicle's free flow speed while the link capacity, C_j , is considered by multiplying lane capacity with total number of lane, l_j (veh/hr), in link j . In this case study, the vehicle's free flow speed and the lane capacity are assumed to be 100km/h and 2,000 veh/hr respectively.

$$T_o^j = \frac{d_j}{100} \quad \text{Eq. (21)}$$

$$C_j = 2000 \cdot l_j \quad \text{Eq. (22)}$$

Notations Used in Travel Time Minimization

$Z_{j,k}$	Total travel time for all link $j = 1, 2, \dots, J$, between an O-D pair within the network when link k under resurfacing (hrs), which is an integral from 0 to q_j (hr)
q_j	Travel flow in link j (veh/hr)
Q	Overall Origin to destination demand (veh)
T_o^j	Travel time between an O-D pair under a free flow speed condition on link j in the network (hr)
C_j	Total link capacity of link j (veh/hr)
d_j	Total distance of link j
l_j	Total number of lane in link j

Table 5: Descriptions of notations used in Travel Time Minimization



Beside the case without any work zone, there are other 5 cases, which assuming one lane is closed due to resurfacing activity on link 1 to 5 accordingly, are considered. The optimal traffic flow distribution would be different among all the cases as the link capacity is changing. In short, there are totally 6 cases would be calculated:

- Case I: No work zone on any link within the network
- Case II: 1 lane out of 2 lanes on link 1 is partially closed for the work zone area
- Case III: 1 lane out of 2 lanes on link 2 is partially closed for the work zone area
- Case IV: 1 lane out of 3 lanes on link 3 is partially closed for the work zone area
- Case V: 1 lane out of 3 lanes on link 4 is partially closed for the work zone area
- Case VI: 1 lane out of 4 lanes on link 5 is partially closed for the work zone area

The procedures in calculating the optimal traffic flow under user equilibrium condition based on the ideas of Bureau of Public Roads (BPR) link congestion function by minimizing the sum of travel time for all link of case without any work zone area on any link across the whole network (Case I) is listed here as an example. Other cases can be calculated in similar approach by substituting corresponding values into Eq. (19) to Eq. (22). MATLAB is used in this study for executing the optimization problem, and the detail of steps in solving Case II to VI can be found in Appendix 1.

Case I: No work zone on any link within the network

The system of equations in Case I can be expressed in the following, each integral part based on BPR congestion function represent the travel time spent in one link, which is an integral from 0 to the corresponding links' flow in order to find the optimal traffic flow distribution across the network. The constraint equation is important for conserving the overall O-D demand, which the overall demand should be equal the sum of traffic flow for all links. The execution of MATLAB is divided into the period of day (daytime and night time) with respect to the O-D pair (A-node and node-B), therefore there are four executions for one single case.



[1] During daytime: O-D demand is 14,000 veh/hr,

A-node:

$$\min_{q_1, q_2, q_3} z =$$

$$\int_0^{q_1} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000}\right)^4\right) dq + \int_0^{q_2} \frac{25}{100} \left(1 + 0.15 \left(\frac{q}{4000}\right)^4\right) dq + \int_0^{q_3} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000}\right)^4\right) dq$$

$$s. t. \quad q_1 + q_2 + q_3 = 14000$$

```
>> clear
>> fn1 = @(q)(1+0.15*(q/4000).^4);
>> fn2 = @(q)(1+0.15*(q/6000).^4);
>> fn3 = @(q)(1+0.15*(q/2000).^4);
>> fun1 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>> fun2 = @(q)integral(@(x)(25/100)*fn1(x),0,q);
>> fun3 = @(q)integral(@(x)(30/100)*fn2(x),0,q);
>> fun = @(x)(fun1(x(1))+fun2(x(2))+fun3(x(3)));
>> Aeq = [1,1,1];
>> beq = 14000;
>> x0 = 14000/3*Aeq;
>> lb = [0,0,0];
>> z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

∴ The flow distribution from origin A to node during daytime is $\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 3657.6 \\ 4856.1 \\ 5486.3 \end{bmatrix}$

node-B:

$$\min_{q_4, q_5} z = \int_0^{q_4} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000}\right)^4\right) dq + \int_0^{q_5} \frac{40}{100} \left(1 + 0.15 \left(\frac{q}{8000}\right)^4\right) dq$$

$$s. t. \quad q_4 + q_5 = 14000$$

```
>> clear
>>fn1 = @(q)(1+0.15*(q/6000).^4);
>>fn2 = @(q)(1+0.15*(q/8000).^4);
>>fun1 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>>fun2 = @(q)integral(@(x)(40/100)*fn2(x),0,q);
>>fun = @(x)(fun1(x(1))+fun2(x(2)));
>>Aeq = [1,1];
>>beq = 14000;
>>x0 = 14000/2*Aeq;
>>lb = [0,0];
>>z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

∴ The flow distribution from node to destination B during daytime is $\begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 7713 \\ 6287 \end{bmatrix}$



[2] During night time: O-D demand is 1,400 veh/hr,

A-node:

$$\min_{q_1, q_2, q_3} z =$$

$$\int_0^{q_1} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_2} \frac{25}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_3} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq$$

$$s.t. \quad q_1 + q_2 + q_3 = 1400$$

```
>> clear
>> fn1 = @(q)(1+0.15*(q/4000).^4);
>> fn2 = @(q)(1+0.15*(q/6000).^4);
>> fun1 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>> fun2 = @(q)integral(@(x)(25/100)*fn1(x),0,q);
>> fun3 = @(q)integral(@(x)(30/100)*fn2(x),0,q);
>> fun = @(x)(fun1(x(1))+fun2(x(2))+fun3(x(3)));
>> Aeq = [1,1,1];
>> beq = 1400;
>> x0 = 1400/3*Aeq;
>> lb = [0,0,0];
>> z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

∴ The flow distribution from origin A to node during night time is $\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1400 \\ 0 \end{bmatrix}$

node-B:

$$\min_{q_4, q_5} z = \int_0^{q_4} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq + \int_0^{q_5} \frac{40}{100} \left(1 + 0.15 \left(\frac{q}{8000} \right)^4 \right) dq$$

$$s.t. \quad q_4 + q_5 = 1400$$

```
>> clear
>>fn1 = @(q)(1+0.15*(q/6000).^4);
>>fn2 = @(q)(1+0.15*(q/8000).^4);
>>fun1 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>>fun2 = @(q)integral(@(x)(40/100)*fn2(x),0,q);
>>fun = @(x)(fun1(x(1))+fun2(x(2)));
>>Aeq = [1,1];
>>beq = 1400;
>>x0 = 1400/2*Aeq;
>>lb = [0,0];
>>z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

∴ The flow distribution from node to destination B during night time is $\begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 1400 \\ 0 \end{bmatrix}$



Referring the Eq. (19) to Eq. (22) and the example of Case I showed above, other cases can be solved by MATLAB with the corresponding system of equations, and the summary of optimal traffic flow distribution for each link grouped by the period of day is listed in the table below.

CASE / LINK			1	2	3	4	5
I	Capacity (veh/hr)		4000	4000	6000	6000	8000
	D	Flow (veh/hr)	3657.6	4856.1	5486.3	7713	6287
	N		0	1400	0	1400	0
II	Capacity (veh/hr)		2000 [#]	4000	6000	6000	8000
	D	Flow (veh/hr)	2180.7	5277.2	6542.1	7713	6287
	N		0	1400	0	1400	0
III	Capacity (veh/hr)		4000	2000 [#]	6000	6000	8000
	D	Flow (veh/hr)	4522.3	2694.4	6783.4	7713	6287
	N		0	1400	0	1400	0
IV	Capacity (veh/hr)		4000	4000	4000 [#]	6000	8000
	D	Flow (veh/hr)	4361.4	5277.2	4361.4	7713	6287
	N		0	1400	0	1400	0
V	Capacity (veh/hr)		4000	4000	6000	4000 [#]	8000
	D	Flow (veh/hr)	3657.6	4856.1	5486.3	5600.4	8399.6
	N		0	1400	0	1400	0
VI	Capacity (veh/hr)		4000	4000	6000	6000	6000 [#]
	D	Flow (veh/hr)	3657.6	4856.1	5486.3	8159.2	5840.8
	N		0	1400	0	1400	0
D: Daytime, N: Night Time							
[#] Link is partially closed due to resurfacing							

Table 6: Computation result of traffic flow distribution by user equilibrium flow distribution with BPR congestion function

4.3 User Cost & Rehabilitation Cost

Rehabilitation cost and user additional operational cost due to the pavement condition such as more fuel is consumed to support the vehicles for a worse pavement condition are considered here. In general, the pavement condition is indicated by the International Roughness Index (IRI) which worse condition contributes a higher value of IRI. The IRI increase exponentially with time, while average operational cost per vehicles is likely proportional to the International Roughness Index (IRI). The two related graphs are sketched below.

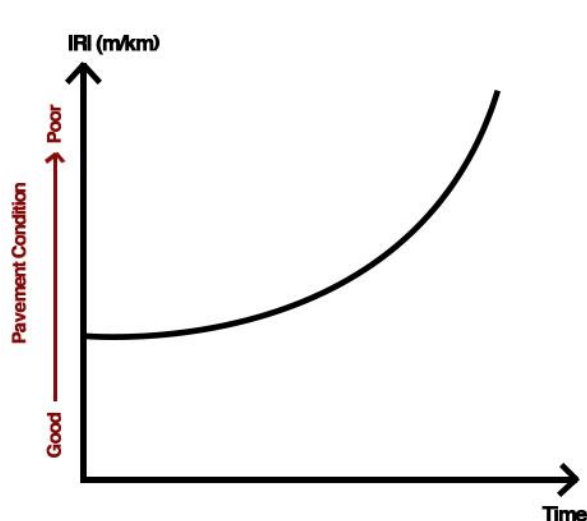


Figure 4: Relationship between pavement condition and time

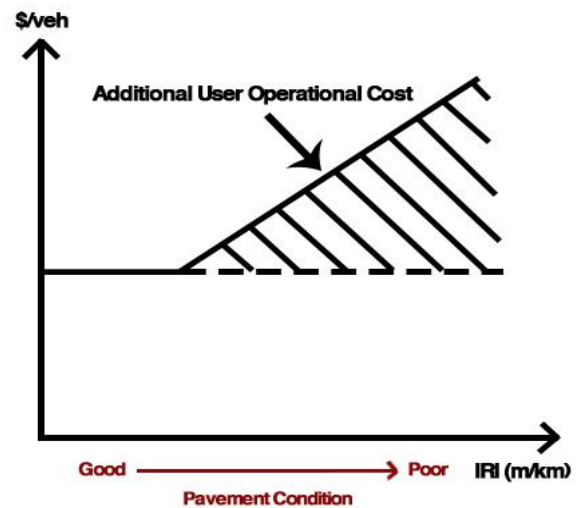


Figure 5: Relationship between average user operational cost and pavement condition

The rate of deterioration of pavement increase with time as shown in Figure 4, while the average user operational cost increase almost at a constant rate with the pavement condition as shown in Figure 5. Therefore, rehabilitation activity of pavement should be carried out at an appropriate time to prevent unreasonable additional user operational cost.



In fact, the user cost of travelling includes the relevant cost in term of travel time and travel distance, and the operational cost with respect to pavement roughness. The travel cost in term of time and distance which within an allowable pavement condition is assumed to be zero as they are the basic cost for travelling and not much influenced by pavement condition while the additional travel cost result from the rehabilitation activity will be discussed later in this section, denoted H_j . Correspondingly, the basic operational cost is also assumed to be zero while the additional user operational cost is taken into account in this research, which above the acceptable user operational cost.

Referring to the previous section of formulation, the discounted user operational cost in this simple case study depends on the pavement condition just after resurfacing, s_j^+ (IRI, m/km), traffic flow, q_j (veh/hr), in link j and the relevant vehicle operating cost corresponding to IRI, c (\$/veh/IRI/ km), where $\bar{q}^j$ refers to the annual average traffic on link j (veh/yr) when there is no any work zone area.

$$U_j = d_j \int_0^{\tau_j} 365.24 \cdot \sum_{t=0}^{23} q_t^j \cdot c \cdot s_j^+ \cdot e^{(b-r)t} dt = d_j \int_0^{\tau_j} \bar{q}^j \cdot c \cdot s_j^+ \cdot e^{(b-r)t} dt \quad \text{Eq. (23)}$$

The additional travel cost, H_j , which is either expressed in distance or time, is in conjunction with Vehicle Hours Travelled (VHT) and Vehicle Miles Travelled (VMT). In order to avoid enormous traffic disturbances, rehabilitation activity will not be carried out for the whole lane at once which will cause the excessive impact on the current traffic flow. In practical, the lane which rehabilitated will be divided into several sections and be resurfaced section by section, where each section is well known as approximately 1.5km long from empirical research. Referring to the formula stated in the previous section, the additional travel cost on link j is expressed in below.

$$H_j = (\Delta VHT_j \cdot u_1 + \Delta VMT_j \cdot u_2) \cdot \frac{d_j}{1.5}$$



Recapping from the previous section, Vehicle Hours Travelled (VHT) is the total vehicle hours expended traveling on the roadway in the network system in an hour during the pavement rehabilitation period while Vehicle Miles Travelled (VMT) is the measurement of the total kilometre travelled by all vehicles in an hour within the network system during the pavement rehabilitation period. The VHTs and VMTs for all links due to roadway closure are calculated by following equations.

$$\begin{aligned} VHT_j &= \sum_{j=1}^J \sum_{t=0}^{D_k-1} VHT_j^k = \sum_{k=1}^J \sum_{t=0}^{D_k-1} T_t^{j,k} \cdot q_t^{j,k} \\ &= \sum_{j=1}^J 2 \text{ days} \cdot 12 \text{ hrs/day} \cdot T_{day}^{j,k} \cdot q_{day}^{j,k} + 2 \text{ days} \cdot 12 \text{ hrs/day} \cdot T_{nig}^{j,k} \cdot q_{nig}^{j,k} \end{aligned}$$

In this preliminary simple network model, it is assumed that the rapid resurfacing activity normally takes two days to accomplish, and the calculation is divided into daytime and night time because of the different traffic flow in different period. Therefore, VHT_j should be considered separately on the period of the day by referring Eq. (7).

Similarly, Vehicle Miles Travelled (VMT) should also be calculated separately during daytime and night time by referring Eq. (8).

$$\begin{aligned} VMT_j &= \sum_{j=1}^J \sum_{t=0}^{D_k-1} VMT_j^k = \sum_{j=1}^J \sum_{t=0}^{D_k-1} d_j \cdot q_t^{j,k} \\ &= 2 \text{ days} \cdot 12 \text{ hrs/day} \cdot q_{day}^{j,k} \cdot d_j + 2 \text{ days} \cdot 12 \text{ hrs/day} \cdot q_{nig}^{j,k} \cdot v_j \end{aligned}$$



On the other hand, the rehabilitation cost, denoted A_j , depends on the resurfacing intensity and the number of lanes required for rehabilitation activity. Besides the sectional resurfacing feature which is discussed above, it is also assumed that only truck and heavy good vehicles will contribute to the pavement deterioration which almost uses the nearside lane in this preliminary study, hence it means there is only one lane need to be resurfaced due to pavement deterioration. Referring the formulas stated in the previous section, rehabilitation cost on link j is expressed in below.

$$A_j = [(m_1 w_j + m_2) \cdot 1.5 \cdot l_j] \cdot \frac{d_j}{1.5}$$

Due to the sectional feature of the additional travel cost and the rehabilitation cost, it is necessary to multiply the number of sections, which is defined as the total distance of lane divided by the section length, in order to generate the cost of resurfacing for a single whole lane.

In order to calculate the cost listed above, those parameters value required which can be found from empirical research, and the required values for this simple case study are listed below.

Descriptions	Parameter	Value (\$)
Rehabilitation cost (\$/lane · km · inch)	m_1	10,491
Rehabilitation cost (\$/lane · km)	m_2	33,012
User cost per unit travel time for all purposes (\$/hr · vehicle)	u_1	12.5
User cost per unit distance (\$/km · vehicle)	u_2	2.33

Table 7: Value of parameters required for calculating additional travel cost and rehabilitation cost



The additional travel cost is calculated based on the travel time and link distance of each link during the different period of a day. Therefore, the corresponding travel time should be generated first as the input for the additional travel cost function. The travel time can be calculated by the BPR link congestion function as follow:

$$T_j = T_o^j \left(1 + 0.15 \left(\frac{q}{C_j} \right)^4 \right) \quad \text{Eq. (24)}$$

Notations Used in BPR Link Congestion Function

T_j	Travel time between an O-D pair on link j in the network (hr)
T_o^j	Travel time between an O-D pair under a free flow speed condition on link j in the network (hr)
q	Overall Origin to destination demand (veh/hr)
C_j	Total Link capacity (veh/hr)

Table 8: Descriptions of notations used in Travel Time Minimization

The table below shows the computation result of travel time for each link grouped by cases which is generated by Microsoft Excel.

j	Daytime					Night time				
	T (hr)	d (km)	q	$T * q$	$d * q$	T (hr)	d (km)	q	$T * q$	$d * q$
CASE I										
1	0.33146	30	3657.6	1212.35	109728	0.3	30	0	0	0
2	0.33146	25	4856.1	1609.60	121403	0.25056	25	1400	350.788	35000
3	0.33146	30	5486.3	1818.48	164589	0.3	30	0	0	0
4	0.42289	30	7713	3261.72	231390	0.30013	30	1400	420.187	42000
5	0.42289	40	6287	2658.68	251480	0.4	40	0	0	0
CASE II										
1	0.36360	30	2180.7	792.909	65421	0.3	30	0	0	0
2	0.36361	25	5277.2	1918.83	131930	0.25056	25	1400	350.788	35000
3	0.36360	30	6542.1	2378.73	196263	0.3	30	0	0	0
4	0.42289	30	7713	3261.72	231390	0.30013	30	1400	420.187	42000
5	0.42289	40	6287	2658.68	251480	0.4	40	0	0	0



CASE III										
1	0.37352	30	4522.3	1689.17	135669	0.3	30	0	0	0
2	0.37353	25	2694.4	1006.43	67360	0.25900	25	1400	362.605	35000
3	0.37352	30	6783.4	2533.73	203502	0.3	30	0	0	0
4	0.42289	30	7713	3261.72	231390	0.30013	30	1400	420.187	42000
5	0.42289	40	6287	2658.68	251480	0.4	40	0	0	0
CASE IV										
1	0.36360	30	4361.4	1585.82	130842	0.3	30	0	0	0
2	0.36361	25	5277.2	1918.83	131930	0.25056	25	1400	350.788	35000
3	0.36330	30	4361.4	1585.82	130842	0.3	30	0	0	0
4	0.42289	30	7713	3261.72	231390	0.30013	30	1400	420.187	42000
5	0.42289	40	6287	2658.68	251480	0.4	40	0	0	0
CASE V										
1	0.33146	30	3657.6	1212.35	109728	0.3	30	0	0	0
2	0.33146	25	4856.1	1609.60	121403	0.25056	25	1400	350.788	35000
3	0.33146	30	5486.3	1818.48	164589	0.3	30	0	0	0
4	0.47292	30	5600.4	2648.55	168012	0.30068	30	1400	420.945	42000
5	0.47292	40	8399.6	3972.31	335984	0.4	40	0	0	0
CASE VI										
1	0.33146	30	3657.6	1212.35	109728	0.3	30	0	0	0
2	0.33146	25	4856.1	1609.60	121403	0.25056	25	1400	350.788	35000
3	0.33146	30	5486.3	1818.48	164589	0.3	30	0	0	0
4	0.45389	30	8159.2	3703.35	244776	0.30013	30	1400	420.187	42000
5	0.45388	30	5840.8	2651.03	233632	0.4	40	0	0	0

Table 9: Computation result of travel time and vehicle speed of each link in different cases



Using the parameters and result listed in Table 6 to Table 9, the additional travel cost in this preliminary network model can be calculated as below.

	$\sum \text{VHT (hr)}$	$\sum \text{VMT (km)}$	$\sum \Delta \text{VHT (hr)}$	$\sum \Delta \text{VMT (km)}$	$\sum \text{H ($)}$
I	135981.57	11467074	-	-	-
II	141382.03	11441808	5400.45	-25266	172717.775
III	143190.23	11596812	7208.66	129738	6539963.15
IV	141382.03	11441808	5400.45	--25266	172717.775
V	144396.19	11720586	8414.62	253512	13917313
VI	141189.24	11413530	5207.66	-53544	-1590979.02

Table 10: Computation result of the additional travel cost and rehabilitation cost

From the above result, it showed a rehabilitation activity contribute an additional vehicle travel time and thus the additional travel cost. Although the vehicle miles travelled in some cases is decreasing and there is a decline in the overall additional travel cost in case 5, the additional travel cost in most of the cases is still increasing. Therefore, it is important to minimize the user cost for a cost-effective rehabilitation activity by finding an optimal time interval and threshold pavement condition for resurfacing.

In fact, the situation in case 5 would be happened in the reality due to the Braess's paradox theory [23], it explains the potential of decreasing traffic congestion when the existing roadway is closing, and the additional travel cost would then be less than that in the normal situation.

However, the huge amount of agency pavement maintenance cost contributes to the overall pavement investment, which includes both user cost and agency cost, it is a good practice to minimize the traffic impact due to work zone area in order to prevent the unnecessary and worthless traffic disturbance.



4.4 Optimal Threshold Pavement Condition and Resurfacing Period

After dealing with the optimal traffic redistribution and the additional travel cost, it is now ready to perform the upper-level problem to find the optimal threshold pavement condition and resurfacing period. The calculation is executed by MATLAB and the full version of data results would be included in Appendix 2.

As the focus of this project is to minimize the user cost, the rehabilitation budget, B , is negligible. Referring to Eq. (3), the segment-level Lagrangian function of segment j is expressed in below.

$$L_j = V_j + \Lambda A_j$$

$$= \left[\frac{e^r - 1}{1 - e^{-r\tau_j}} \right] \left[H_j + \left(d_j \cdot \int_0^{\tau_j} \overline{q^j} \cdot c \cdot s_j^+ \cdot e^{(b-r)t} dt \right) + \Lambda \left((m_1 w_j + m_2) \cdot l_j \cdot d_j \right) \right] \quad \text{Eq. (25)}$$

In order to find the optimal threshold pavement condition, optimality equation proposed by Ouyang and Madanat [7] is taken as the reference in this research with some replaced parameters and additional constraints according to our segment-level Lagrangian function. λ is set in a range of 0 to 100 for finding the optimal solution. In addition, it is assumed that the threshold pavement condition with the IRI in between 1.5 m/km and 4.5 m/km as this is the common acceptable roughness range for a roadway pavement indicated by Sayers et al. [24]. The roughness scale of pavements is shown in the next page.

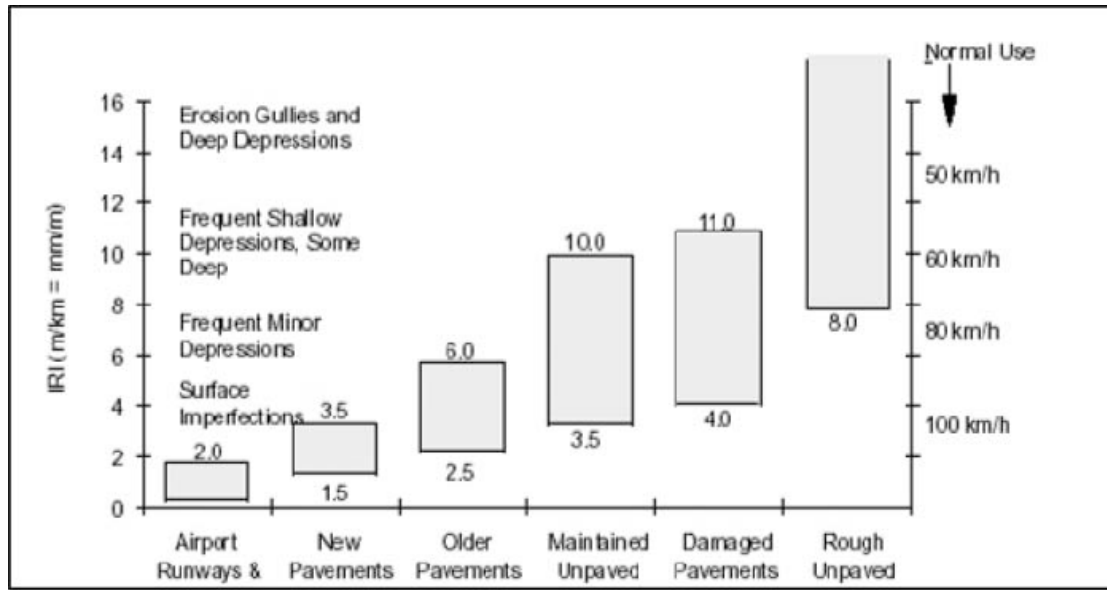


Figure 6: IRI roughness scale [24]

Therefore, the optimal threshold pavement roughness with constraints can be expressed by the equation below.

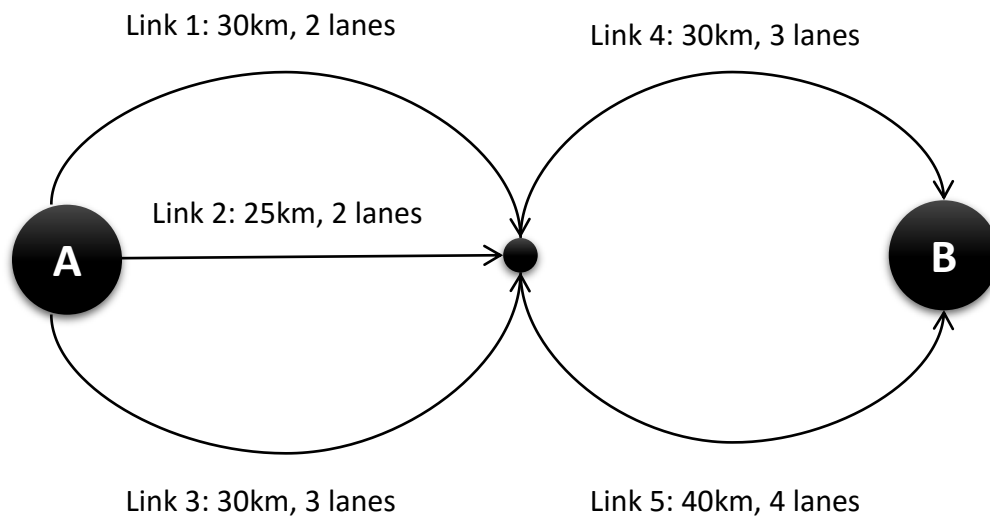
$$x_j^* = \min \left\{ \max \left\{ \frac{r\lambda l_j(m_1 g_3 + m_2) + rH_j}{\bar{q}^j c g_1 + (b - r)\lambda m_1 l_j g_2}, 1.5 \right\}, 4.5 \right\} \quad \text{Eq. (26)}$$

As this research is carried out using an imaginary preliminary network model, some parameters need to be set based on the empirical researches, and the value of required parameters are listed in following:

Descriptions	Parameter	Value
Rehabilitation variable cost (\$/lane · km · inch)	m_1	\$10,491
Rehabilitation fixed cost (\$/lane · km)	m_2	\$33,012
User vehicle operating cost (\$/IRI · km · vehicle)	c	\$0.001785
Resurfacing effectiveness parameter 1	g_1	0.66
Resurfacing effectiveness parameter 2	g_2	0.55
Resurfacing effectiveness parameter 3	g_3	18.3
Discount Rate	r	0.04
Deterioration Rate	b	0.03-0.06

Table 11: Parameters required for calculating optimal threshold pavement roughness in the preliminary network model

As this research treats every road facility to be heterogeneous with regard the deterioration process, the deterioration rate of each link should be difference due to some external factors, such as traffic volume and the weather. Therefore, the deterioration rate is set according to the range in above table for calculation. Referring to the table 6 and Eq. (23), the annual average traffic flow on link j can be calculated for calculating the optimal threshold pavement roughness. The table below shows the deterioration rate and the annual average traffic flow corresponding to each link.



	Link 1	Link 2	Link 3	Link 4	Link 5
b	0.06	0.04	0.04	0.04	0.03
$\bar{q}^j$	16030821.89	27419735.57	24045794.54	39941185.44	27555166.56

Table 12: Deterioration rate and annual average traffic flow on each link for the preliminary network model

The figure below showed part of the result on optimal threshold pavement roughness with λ starting from 0 to 100 based on the preliminary network model. Instead of listing a huge amount of data, this graph showed the abstract of the trend of the optimal threshold pavement roughness with respect to λ among all the links. The text version of data would be included in Appendix 2.

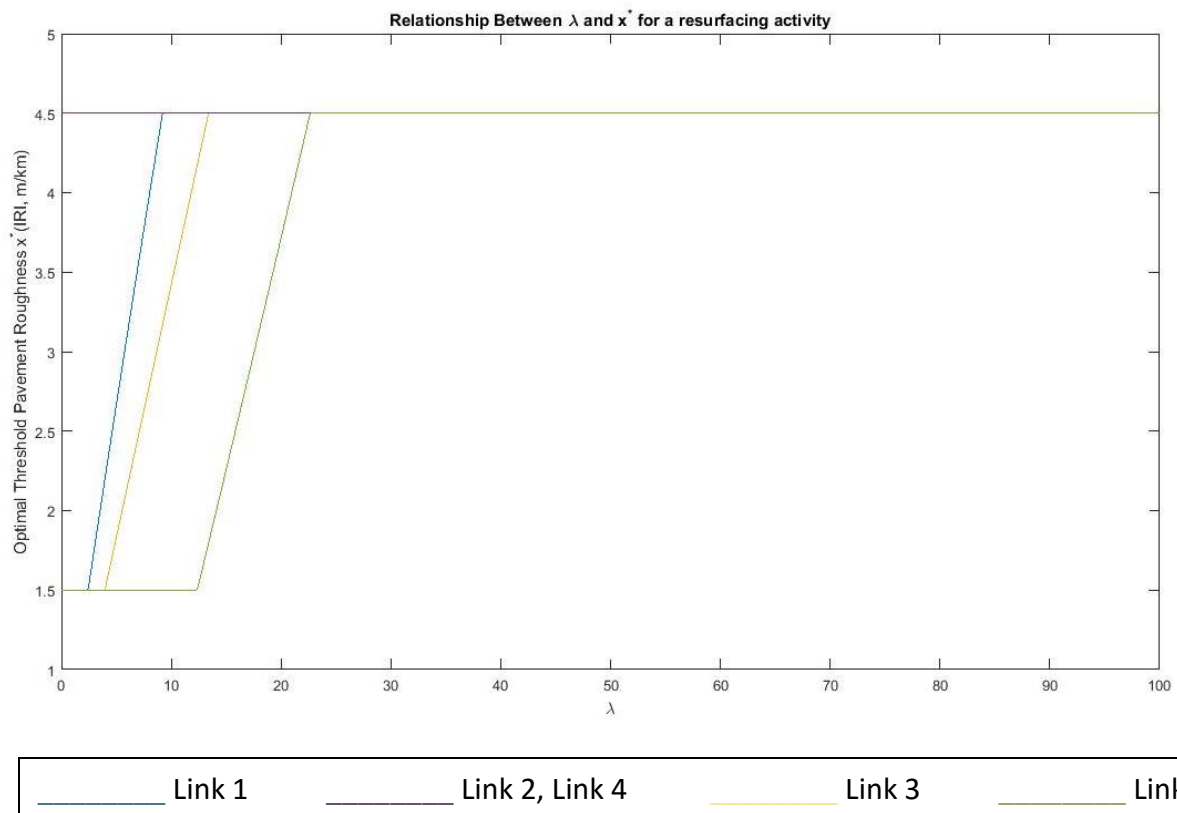


Figure 7: Graph showing the relationship between the optimal threshold pavement roughness and lambda.



Although the relationship between the optimal threshold pavement roughness and λ is meaningless, it is better to include the result into this report to show the features of each link in term of the roughness level. It seems that link 2 and link 4 can be treated as unimportant roadways, which can be omitted in this research, as their threshold pavement roughness are all excess the maximum pavement constraint ($IRI = 4.5m/km$). They probably belong to damage pavement or rough unpaved pavement which has no meaning for undergo resurfacing.

Besides, by referring to the deterioration function Eq. (14) and the resurfacing effectiveness function Eq. (15), the optimal resurfacing period can be obtained by combining these two functions. In general, the coefficient of resurfacing effectiveness (g_1) is 0.66 which mean the pavement can be improved by 0.34 after resurfacing. At the same time, the resurfaced pavement roughness level is restricted not to be below 1 as it is the best achievable roughness. Therefore, the function of the optimal resurfacing period can be expressed as in Eq. (27).

$$\tau_j^* = \frac{\ln \left\{ \frac{x_j^*}{\max[1, 0.34x_j^*]} \right\}}{b_j} \quad \text{Eq. (27)}$$

The corresponding result of Eq. (27) for the preliminary network model used in this report is shown by the following graph.

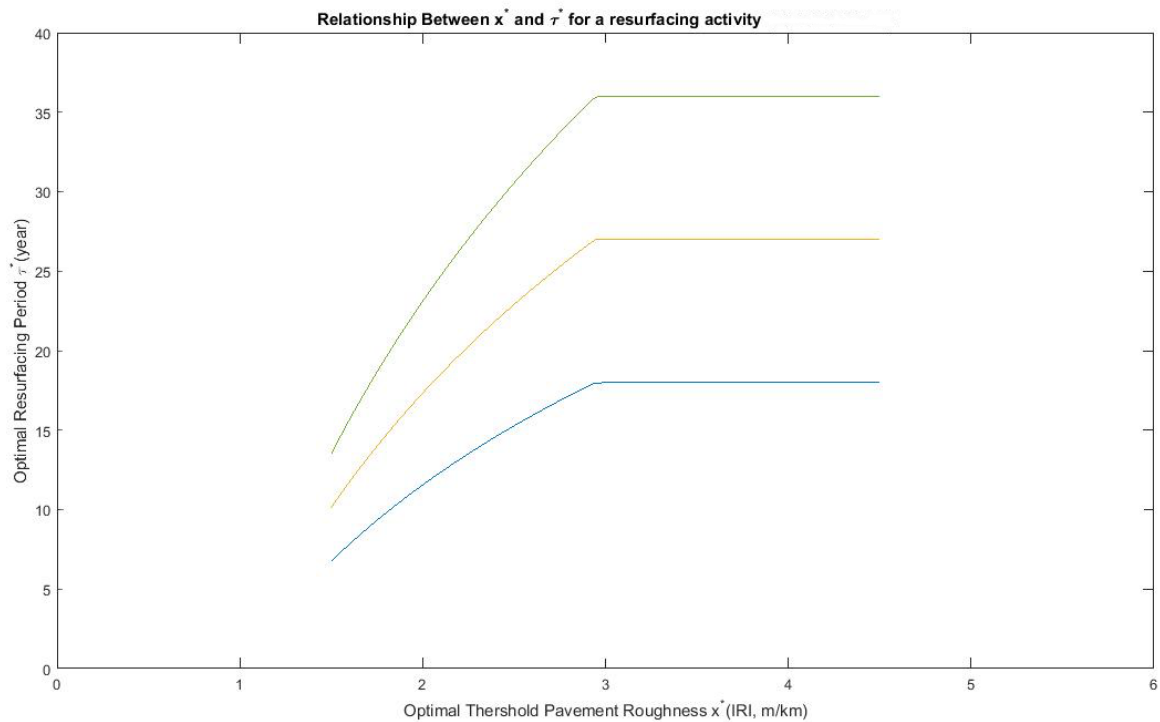


Figure 8: Graph showing the relationship between the optimal threshold pavement roughness and the optimal resurfacing period.



From the above graph, it is obvious that better pavement condition required a shorter resurfacing period, however, the resurfacing period comes to constant when the optimal threshold pavement roughness nearly equivalent to 3 m/km, which indicates there is the least standard of resurfacing period even for those worse condition pavements.



4.5 Overall User Cost and Rehabilitation Cost

Using the result of the optimal threshold pavement roughness of each link, the corresponding optimal resurfacing intensity can also be found in order to recognize the relationship between the user cost and the rehabilitation cost for a single resurfacing activity, and thus to find the minimum overall user cost for a given rehabilitation budget.

According to the Eq. (18), the optimal resurfacing intensity is in a linear relationship with the optimal threshold pavement roughness, and the optimal resurfacing intensity for each link is calculated from the following equation:

$$w_j^* = 0.55 x_j^* + 18.3$$

The intensity changes according to the roughness level and the detail results would also be included in Appendix 2.

On the other hand, the annual average discounted user cost and the annual average discounted rehabilitation cost can be obtained by substituting the optimal threshold pavement roughness and optimal resurfacing period into Eq. (28) and Eq. (29) respectively as below.

$$V^* = \sum_j \left(\frac{e^r - 1}{1 - e^{-r\tau_j^*}} \right) \left(H_j + d_j \int_0^{\tau_j^*} \bar{q}^j c \frac{x_j^*}{3} e^{(b-r)t} dt \right) \quad \text{Eq. (28)}$$

$$A^* = \sum_j \left(\frac{e^r - 1}{1 - e^{-r\tau_j^*}} \right) (d_j l_j (m_1 w_j + m_2)) \quad \text{Eq. (29)}$$

Eq. (28) indicates the annual average discounted user cost varies with the optimal threshold pavement condition, and the relationship between them is shown in the following figure.

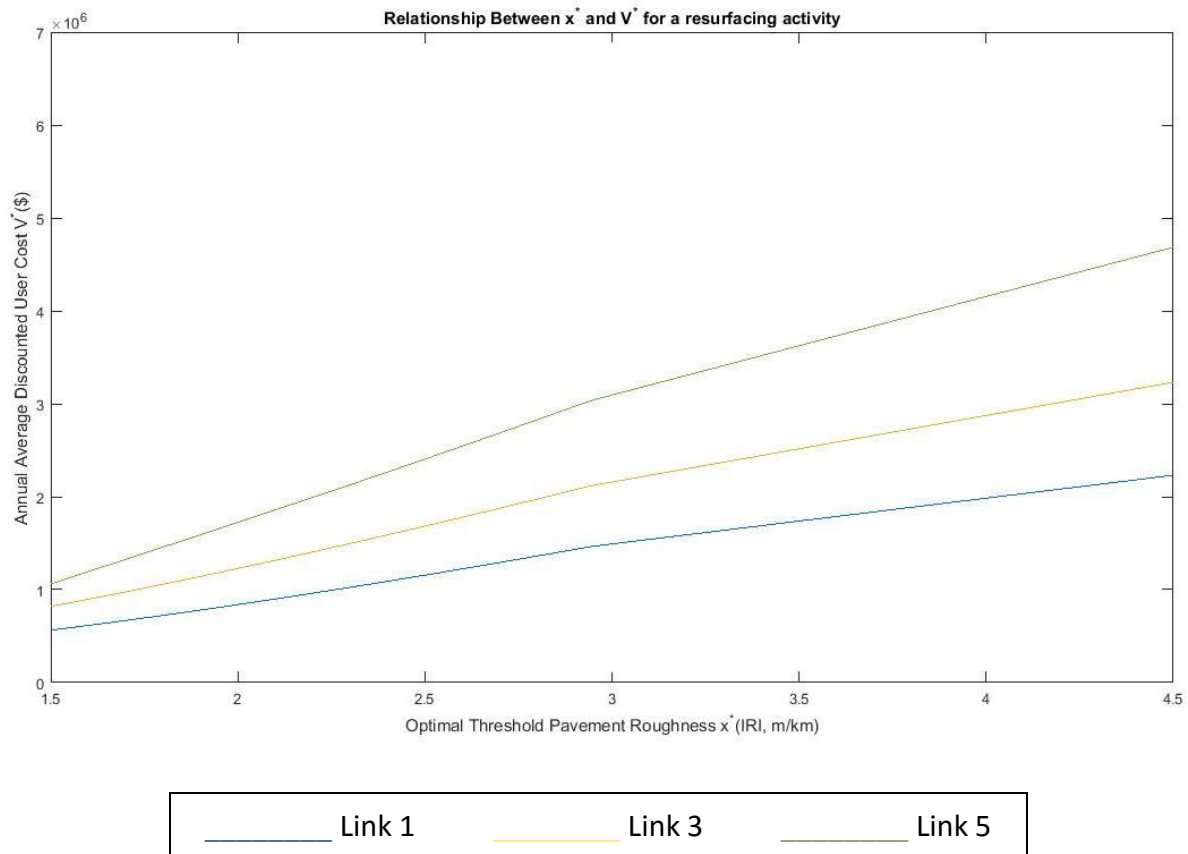


Figure 9: Graph showing the relationship between the optimal threshold pavement roughness and the annual average discount overall user cost.

From the above graph, the annual average discounted user cost increase continuously with the optimal threshold pavement roughness as worse pavement condition impose more user operational cost and thus for the overall user cost. Therefore, it is essential to preserve the pavement serviceability as to reduce the excessive user cost which could be minimized.

The overall relationship between the total annual average discounted rehabilitation cost and the total annual average user cost for all λ is demonstrated by the following figure. In fact, only part of it contributes to this research due to some negligible and meaningless data. From the graph, there are multiple intersection points for some of the given rehabilitation cost, however, only the most minimum point is considered in this case.

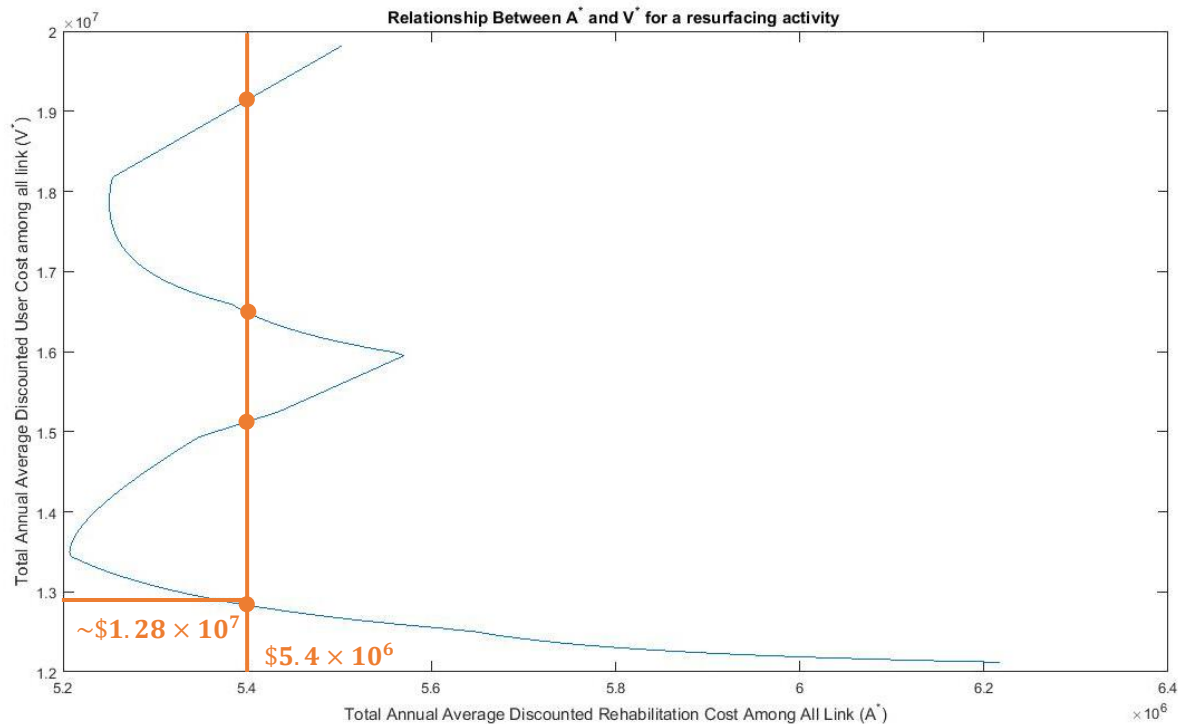


Figure 10: Graph showing the relationship between the total annual average discounted rehabilitation and the total annual average discount overall user cost.

For example, by referring the Figure 10, there are totally four crossing points when the rehabilitation cost equivalent to \$5,400,000 and we would only focus on the most favours user cost, which is around \$1,280,000. As the result, this graph can be simplified as shown in the next page.

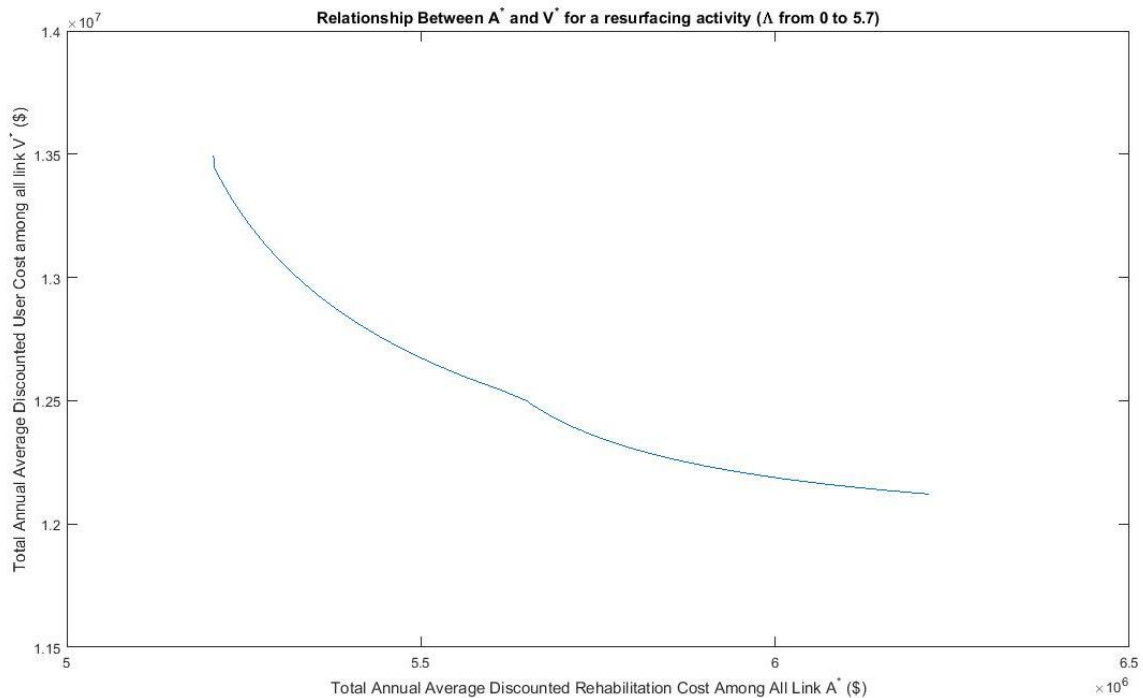


Figure 11: Simplified graph showing the relationship between the total annual average discount rehabilitation cost and the total annual average discount user cost. ($\lambda = 0$ to 5.7)

It is obvious that less user cost resulted from higher amount of budget. The simplified graph showed a kind of Pareto Frontier which is the set of allocation satisfying the Pareto optimality and the Pareto optimality is defined as the optimal allocation of resources which one of those resources cannot be rearranged to make any one of preference to become better without making any one of preference go worse.

Therefore, the optimal threshold pavement condition, resurfacing period and the resurfacing intensity can be figured out for a given rehabilitation budget from the graph corresponding to each link with other related diagrams, such as the relationship between the overall user cost and the threshold roughness level (Figure 9), and the relationship between the threshold pavement condition and the resurfacing period (Figure 8). Case 2, which one out of two lane in link 1 is under resurfacing, is used as example showed below.

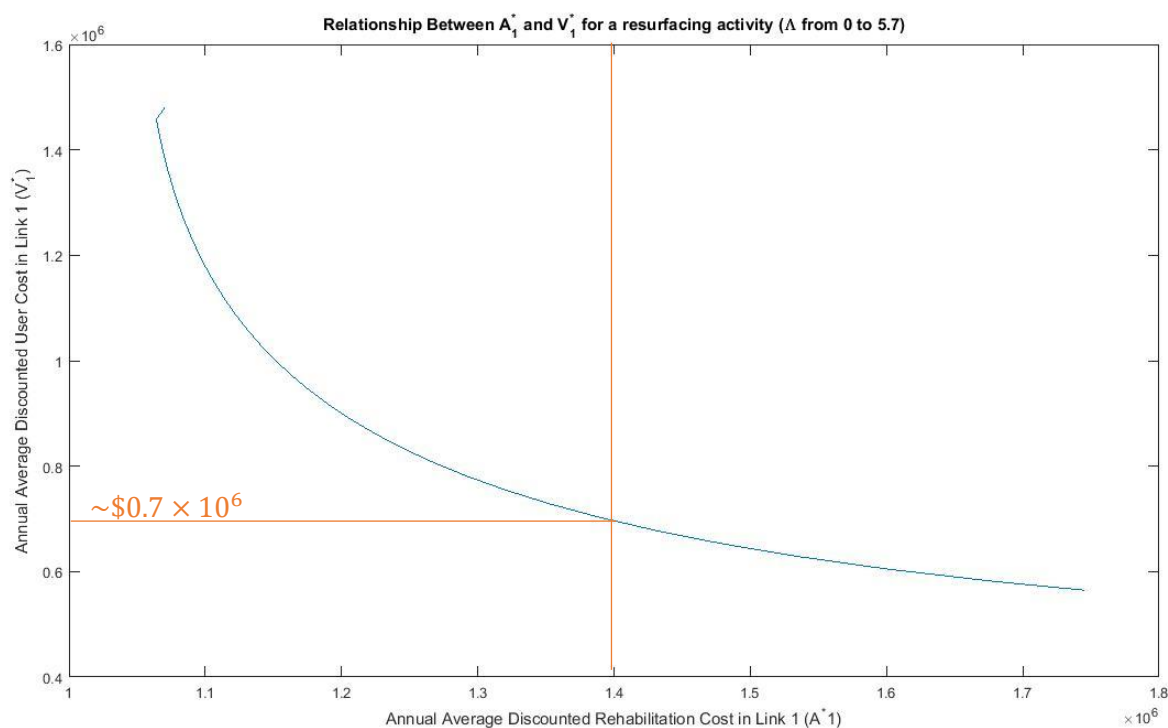


Figure 12: Simplified graph showing the relationship between the annual average discount rehabilitation cost and the annual average discount user cost in link 1. ($\lambda = 0$ to 5.7)

When the rehabilitation budget is assumed to be \$1400000, according to the result, the optimal average discount user cost in link 1 would be around \$700000. Later on, the optimal threshold pavement roughness could be found based on the optimal user cost we found. Hence, from Figure 9, it showed the optimal threshold pavement roughness is approximately equal to 2 m/km. Finally, the optimal resurfacing period would also be found by referring Figure 8.



Chapter 5: Conclusion

5.1 Further Improvement

Although this research has developed a preliminary model for optimal resurfacing planning, there is a room for improvement. This research only considers a single resurfacing activity, however multiple rehabilitation activities should be taken into account in order to make the model be more comprehensive. Besides, a preliminary case study is carried out in this report, which is only imaginal, therefore it is not good enough to deal with the reality due to its complexity. It is suggested to perform a real case study for improvement.

5.2 Summary

To conclude, maintenance and rehabilitation activities are necessary to preserve the pavement serviceability in transportation system which requires occupancy of travel lanes or roadway closure. An appropriate way to quantify traffic disturbances for user cost minimization due to rehabilitation is essential to the pavement management system as it is much serious compared with that of agency cost. Although there are lots of studies have handled similar optimization problem before, most of them are restricted in system level optimization or different from our project scope. Therefore, this report shows the methodology of dealing the user cost minimization due to resurfacing by taking account into both traffic redistribution and transportation network topology through a case study on the preliminary network model.



This report focuses on the preliminary case study with simple network design which aims to perform the optimization process of work zone impacts on traffic within a network in a systematic manner. At the end of this project, formulation and solution method considering transportation network topology and traffic redistribution regarding the user cost optimization problem is executed. The computation result satisfies the Pareto Optimality and it shows the Pareto set which could help to make focused tradeoffs within the related parameters.

In short, it is important and necessary to incorporate the work zone impacts on traffic into network-level pavement management systems as to avoid an excessive but worthless pavement maintenance cost. A preliminary model for optimal resurfacing planning is developed in this research.



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Appendix 1:

Computation of Traffic Flow Distribution by Bureau of Public Roads (BPR) link congestion function

Case II: 1 lane out of 2 lanes on link 1 is partially closed for the work zone area

[1] During day time: O-D demand is 14,000 veh/hr

A-node:

(Total capacity in link 1 is reduced from 4,000 veh/hr to 2,000 veh/hr)

$$\min_{q_1, q_2, q_3} z = \int_0^{q_1} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{2000} \right)^4 \right) dq + \int_0^{q_2} \frac{25}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_3} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq$$

$$s. t. \quad q_1 + q_2 + q_3 = 14000$$

```
>> clear
>> fn1 = @(q)(1+0.15*(q/4000).^4);
>> fn2 = @(q)(1+0.15*(q/6000).^4);
>> fn3 = @(q)(1+0.15*(q/2000).^4);
>> fun1 = @(q)integral(@(x)(30/100)*fn3(x),0,q);
>> fun2 = @(q)integral(@(x)(25/100)*fn1(x),0,q);
>> fun3 = @(q)integral(@(x)(30/100)*fn2(x),0,q);
>> fun = @(x)(fun1(x(1))+fun2(x(2))+fun3(x(3)));
>> Aeq = [1,1,1];
>> beq = 14000;
>> x0 = 14000/3*Aeq;
>> lb = [0,0,0];
>> z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

$$\therefore \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 2180.7 \\ 5277.2 \\ 6542.1 \end{bmatrix}$$

node-B:

(There is no change on capacity in both link 4 and link 5 if link 1 is under resurfacing, the solution should be same as the solution of node-B in Case 1)

$$\min_{q_4, q_5} z = \int_0^{q_4} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq + \int_0^{q_5} \frac{40}{100} \left(1 + 0.15 \left(\frac{q}{8000} \right)^4 \right) dq$$

$$s. t. \quad q_4 + q_5 = 14000$$

$$\therefore \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 7713 \\ 6287 \end{bmatrix}$$



[2] During night time: O-D demand is 1,400 veh/hr

A-node:

$$\min_{q_1, q_2, q_3} z =$$

$$\int_0^{q_1} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{2000}\right)^4\right) dq + \int_0^{q_2} \frac{25}{100} \left(1 + 0.15 \left(\frac{q}{4000}\right)^4\right) dq + \int_0^{q_3} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000}\right)^4\right) dq$$

$$s.t. \quad q_1 + q_2 + q_3 = 1400$$

```
>> clear
>> fn1 = @(q)(1+0.15*(q/4000).^4);
>> fn2 = @(q)(1+0.15*(q/6000).^4);
>> fn3 = @(q)(1+0.15*(q/2000).^4);
>> fun1 = @(q)integral(@(x)(30/100)*fn3(x),0,q);
>> fun2 = @(q)integral(@(x)(25/100)*fn1(x),0,q);
>> fun3 = @(q)integral(@(x)(30/100)*fn2(x),0,q);
>> fun = @(x)(fun1(x(1))+fun2(x(2))+fun3(x(3)));
>> Aeq = [1,1,1];
>> beq = 1400;
>> x0 = 1400/3*Aeq;
>> lb = [0,0,0];
>> z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

$$\therefore \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1400 \\ 0 \end{bmatrix}$$

node-B:

(There is no change on capacity in both link 4 and link 5 if link 1 is under resurfacing, so the solution should be same as the solution of node-B in Case I)

$$\min_{q_4, q_5} z = \int_0^{q_4} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000}\right)^4\right) dq + \int_0^{q_5} \frac{40}{100} \left(1 + 0.15 \left(\frac{q}{8000}\right)^4\right) dq$$

$$s.t. \quad q_4 + q_5 = 1400$$

$$\therefore \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 1400 \\ 0 \end{bmatrix}$$



Case III: 1 lane out of 2 lanes on link 2 is partially closed for the work zone area

[1] During day time: O-D demand is 14,000 veh/hr

A-node:

(Total capacity in link 2 is reduced from 4,000 veh/hr to 2,000 veh/hr)

$$\min_{q_1, q_2, q_3} z =$$

$$\int_0^{q_1} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_2} \frac{25}{100} \left(1 + 0.15 \left(\frac{q}{2000} \right)^4 \right) dq + \int_0^{q_3} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq$$

$$s. t. \quad q_1 + q_2 + q_3 = 14000$$

```
>> clear
>> fn1 = @(q)(1+0.15*(q/4000).^4);
>> fn2 = @(q)(1+0.15*(q/6000).^4);
>> fn3 = @(q)(1+0.15*(q/2000).^4);
>> fun1 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>> fun2 = @(q)integral(@(x)(25/100)*fn3(x),0,q);
>> fun3 = @(q)integral(@(x)(30/100)*fn2(x),0,q);
>> fun = @(x)(fun1(x(1))+fun2(x(2))+fun3(x(3)));
>> Aeq = [1,1,1];
>> beq = 14000;
>> x0 = 14000/3*Aeq;
>> lb = [0,0,0];
>> z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

$$\therefore \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 4522.3 \\ 2694.4 \\ 6783.4 \end{bmatrix}$$

node-B:

(There is no change on capacity in both link 4 and link 5 if link 2 is under resurfacing, the solution should be same as the solution of node-B in Case I)

$$\min_{q_4, q_5} z = \int_0^{q_4} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq + \int_0^{q_5} \frac{40}{100} \left(1 + 0.15 \left(\frac{q}{8000} \right)^4 \right) dq$$

$$s. t. \quad q_4 + q_5 = 14000$$

$$\therefore \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 7713 \\ 6287 \end{bmatrix}$$



[2] During night time: O-D demand is 1,400 veh/hr

A-node:

(Total capacity in link 2 is reduced from 4,000 veh/hr to 2,000 veh/hr)

$$\min_{q_1, q_2, q_3} z =$$

$$\int_0^{q_1} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_2} \frac{25}{100} \left(1 + 0.15 \left(\frac{q}{2000} \right)^4 \right) dq + \int_0^{q_3} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq$$

$$s.t. \quad q_1 + q_2 + q_3 = 1400$$

```
>> clear
>> fn1 = @(q)(1+0.15*(q/4000).^4);
>> fn2 = @(q)(1+0.15*(q/6000).^4);
>> fn3 = @(q)(1+0.15*(q/2000).^4);
>> fun1 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>> fun2 = @(q)integral(@(x)(25/100)*fn3(x),0,q);
>> fun3 = @(q)integral(@(x)(30/100)*fn2(x),0,q);
>> fun = @(x)(fun1(x(1))+fun2(x(2))+fun3(x(3)));
>> Aeq = [1,1,1];
>> beq = 1400;
>> x0 = 1400/3*Aeq;
>> lb = [0,0,0];
>> z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

$$\therefore \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1400 \\ 0 \end{bmatrix}$$

node-B:

(There is no change on capacity in both link 4 and link 5 if link 2 is under resurfacing, the solution should be same as the solution of node-B in Case I)

$$\min_{q_4, q_5} z = \int_0^{q_4} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq + \int_0^{q_5} \frac{40}{100} \left(1 + 0.15 \left(\frac{q}{8000} \right)^4 \right) dq$$

$$s.t. \quad q_4 + q_5 = 1400$$

$$\therefore \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 1400 \\ 0 \end{bmatrix}$$



Case IV: 1 lane out of 3 lanes on link 3 is partially closed for the work zone area

[1] During day time: O-D demand is 14,000 veh/hr

A-node:

(Total capacity in link 3 is reduced from 6,000 veh/hr to 4,000 veh/hr)

$$\min_{q_1, q_2, q_3} z =$$

$$\int_0^{q_1} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_2} \frac{25}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_3} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq$$

$$s. t. \quad q_1 + q_2 + q_3 = 14000$$

```
>> clear
>> fn1 = @(q)(1+0.15*(q/4000).^4);
>> fun1 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>> fun2 = @(q)integral(@(x)(25/100)*fn1(x),0,q);
>> fun3 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>> fun = @(x)(fun1(x(1))+fun2(x(2))+fun3(x(3)));
>> Aeq = [1,1,1];
>> beq = 14000;
>> x0 = 14000/3*Aeq;
>> lb = [0,0,0];
>> z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

$$\therefore \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 4361.4 \\ 5277.2 \\ 4361.4 \end{bmatrix}$$

node-B:

(There is no change on capacity in both link 4 and link 5 if link 3 is under resurfacing, the solution should be same as the solution of node-B in Case I)

$$\min_{q_4, q_5} z = \int_0^{q_4} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq + \int_0^{q_5} \frac{40}{100} \left(1 + 0.15 \left(\frac{q}{8000} \right)^4 \right) dq$$

$$s. t. \quad q_4 + q_5 = 14000$$

$$\therefore \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 7713 \\ 6287 \end{bmatrix}$$



[2] During night time: O-D demand is 1,400 veh/hr

A-node:

(Total capacity in link 3 is reduced from 6,000 veh/hr to 4,000 veh/hr)

$$\min_{q_1, q_2, q_3} z =$$

$$\int_0^{q_1} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000}\right)^4\right) dq + \int_0^{q_2} \frac{25}{100} \left(1 + 0.15 \left(\frac{q}{4000}\right)^4\right) dq + \int_0^{q_3} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000}\right)^4\right) dq$$

$$s.t. \quad q_1 + q_2 + q_3 = 1400$$

```
>> clear
>> fn1 = @(q)(1+0.15*(q/4000).^4);
>> fun1 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>> fun2 = @(q)integral(@(x)(25/100)*fn1(x),0,q);
>> fun3 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>> fun = @(x)(fun1(x(1))+fun2(x(2))+fun3(x(3)));
>> Aeq = [1,1,1];
>> beq = 1400;
>> x0 = 1400/3*Aeq;
>> lb = [0,0,0];
>> z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

$$\therefore \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1400 \\ 0 \end{bmatrix}$$

node-B:

(There is no change on capacity in both link 4 and link 5 if link 3 is under resurfacing, so the solution should be same as the solution of node-B in Case I)

$$\min_{q_4, q_5} z = \int_0^{q_4} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000}\right)^4\right) dq + \int_0^{q_5} \frac{40}{100} \left(1 + 0.15 \left(\frac{q}{8000}\right)^4\right) dq$$

$$s.t. \quad q_4 + q_5 = 1400$$

$$\therefore \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 1400 \\ 0 \end{bmatrix}$$



Case V: 1 lane out of 3 lanes on link 4 is partially closed for the work zone area

[1] During day time: O-D demand is 14,000 veh/hr

A-node:

(There is no change on capacity in all link 1, link 2 and link 3 if link 4 is under resurfacing, so the solution should be same as the solution of A-node in Case I)

$$\min_{q_1, q_2, q_3} z = \int_0^{q_1} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_2} \frac{25}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_3} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq$$

$$s. t. \quad q_1 + q_2 + q_3 = 14000$$

$$\therefore \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 3657.6 \\ 4856.1 \\ 5486.3 \end{bmatrix}$$

node-B:

(Total capacity in link 4 is reduced from 6,000 veh/hr to 4,000 veh/hr)

$$\min_{q_4, q_5} z = \int_0^{q_4} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_5} \frac{40}{100} \left(1 + 0.15 \left(\frac{q}{8000} \right)^4 \right) dq$$

$$s. t. \quad q_4 + q_5 = 14000$$

```
>> clear
>> fn1 = @(q)(1+0.15*(q/8000).^4);
>> fn2 = @(q)(1+0.15*(q/4000).^4);
>> fun1 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>> fun2 = @(q)integral(@(x)(40/100)*fn2(x),0,q);
>> fun = @(x)(fun1(x(1))+fun2(x(2)));
>> Aeq = [1,1];
>> beq = 14000;
>> x0 = 14000/2*Aeq;
>> lb = [0,0];
>> z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

$$\therefore \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 5600.4 \\ 8399.6 \end{bmatrix}$$



[2] During night time: O-D demand is 1,400 veh/hr

A-node:

(There is no change on capacity in all link 1, link 2 and link 3 if link 4 is under resurfacing, so the solution should be same as the solution of A-node in Case 1)

$$\min_{q_1, q_2, q_3} z =$$

$$\int_0^{q_1} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_2} \frac{25}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_3} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq$$

$$s.t. \quad q_1 + q_2 + q_3 = 1400$$

$$\therefore \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1400 \\ 0 \end{bmatrix}$$

node-B:

(Total capacity in link 4 is reduced from 6,000 veh/hr to 4,000 veh/hr)

$$\min_{q_4, q_5} z = \int_0^{q_4} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_5} \frac{40}{100} \left(1 + 0.15 \left(\frac{q}{8000} \right)^4 \right) dq$$

$$s.t. \quad q_4 + q_5 = 1400$$

```
>> clear
>>fn1 = @(q)(1+0.15*(q/8000).^4);
>>fn2 = @(q)(1+0.15*(q/4000).^4);
>>fun1 = @(q)integral(@(x)(30/100)*fn2(x),0,q);
>>fun2 = @(q)integral(@(x)(40/100)*fn1(x),0,q);
>>fun = @(x)(fun1(x(1))+fun2(x(2)));
>>Aeq = [1,1];
>>beq = 1400;
>>x0 = 1400/2*Aeq;
>>lb = [0,0];
>>z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

$$\therefore \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 1400 \\ 0 \end{bmatrix}$$



Case VI: 1 lane out of 4 lanes on link 5 is partially closed for the work zone area

[1] During day time: O-D demand is 14,000 veh/hr

A-node:

(There is no change on capacity in all link 1, link 2 and link 3 if link 5 is under resurfacing, so the solution should be same as the solution of A-node in Case I)

$$\min_{q_1, q_2, q_3} z =$$

$$\int_0^{q_1} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000}\right)^4\right) dq + \int_0^{q_2} \frac{25}{100} \left(1 + 0.15 \left(\frac{q}{4000}\right)^4\right) dq + \int_0^{q_3} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000}\right)^4\right) dq$$

$$s. t. \quad q_1 + q_2 + q_3 = 14000$$

$$\therefore \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 3657.6 \\ 4856.1 \\ 5486.3 \end{bmatrix}$$

node-B:

(Total capacity in link 5 is reduced from 8,000 veh/hr to 6,000 veh/hr)

$$\min_{q_4, q_5} z = \int_0^{q_4} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000}\right)^4\right) dq + \int_0^{q_5} \frac{40}{100} \left(1 + 0.15 \left(\frac{q}{6000}\right)^4\right) dq$$

$$s. t. \quad q_4 + q_5 = 14000$$

Solving by Matlab,

```
>> clear
>>fn1 = @(q)(1+0.15*(q/6000).^4);
>>fun1 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>>fun2 = @(q)integral(@(x)(40/100)*fn1(x),0,q);
>>fun = @(x)(fun1(x(1))+fun2(x(2)));
>>Aeq = [1,1];
>>beq = 14000;
>>x0 = 14000/2*Aeq;
>>lb = [0,0];
>>z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

$$\therefore \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 8159.2 \\ 5840.8 \end{bmatrix}$$



[2] During night time: O-D demand is 1,400 veh/hr

A-node:

(There is no change on capacity in all link 1, link 2 and link 3 if link 5 is under resurfacing, so the solution should be same as the solution of A-node in Case I)

$$\min_{q_1, q_2, q_3} z =$$

$$\int_0^{q_1} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_2} \frac{25}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq + \int_0^{q_3} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{4000} \right)^4 \right) dq$$

$$s.t. \quad q_1 + q_2 + q_3 = 1400$$

$$\therefore \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1400 \\ 0 \end{bmatrix}$$

node-B:

(Total capacity in link 5 is reduced from 8,000 veh/hr to 6,000 veh/hr)

$$\min_{q_4, q_5} z = \int_0^{q_4} \frac{30}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq + \int_0^{q_5} \frac{40}{100} \left(1 + 0.15 \left(\frac{q}{6000} \right)^4 \right) dq$$

$$s.t. \quad q_4 + q_5 = 1400$$

```
>> clear
>>fn1 = @(q)(1+0.15*(q/6000).^4);
>>fun1 = @(q)integral(@(x)(30/100)*fn1(x),0,q);
>>fun2 = @(q)integral(@(x)(40/100)*fn1(x),0,q);
>>fun = @(x)(fun1(x(1))+fun2(x(2)));
>>Aeq = [1,1];
>>beq = 1400;
>>x0 = 1400/2*Aeq;
>>lb = [0,0];
>>z = fmincon(fun,x0,[],[],Aeq,beq,lb)
```

$$\therefore \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 1400 \\ 0 \end{bmatrix}$$



Appendix 2: Full Version of data

Please refer to the Microsoft Excel document embed below for the details.

